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Hybrid control of switched LFT uncertain systems with time-varying input delays

Chengzhi Yuan[a], Yan Gu[b] and Wei Zeng[c]

[a]Department of Mechanical, Industrial and Systems Engineering, University of Rhode Island, Kingston, RI, USA; [b]Department of Mechanical Engineering, University of Massachusetts Lowell, Lowell, MA, USA; [c]School of Mechanical and Electrical Engineering, Longyan University, Longyan, People's Republic of China

ABSTRACT

This paper addresses the problem of hybrid control for a class of switched uncertain systems. The switched system under consideration is subject to structured uncertain dynamics in a linear fractional transformation (LFT) form and time-varying input delays. A novel hybrid controller is proposed, which consists of three major components: the integral quadratic constraint (IQC) dynamics, the continuous dynamics, and the jump dynamics. The IQC dynamics are developed by leveraging methodologies from robust control theory and are utilised to address the effects of time-varying input delays. The continuous dynamics are structured by feeding back not only measurement outputs but also some system's internal signals. The jump dynamics enforce a jump (update/reset) at every switching time instant for the states of both IQC dynamics and continuous dynamics. Based on this, robust stability of the overall hybrid closed-loop system is established under the average dwell time framework with multiple Lyapunov functions. Moreover, the associated control synthesis conditions are fully characterised as linear matrix inequalities, which can be solved efficiently. An application example on regulation of a nonlinear switched electronic circuit system has been used to demonstrate effectiveness and usefulness of the proposed approach.

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Switched systems; hybrid switching and impulsive control; time-varying input delay; linear fractional transformation; linear matrix inequality

1. Introduction

As a typical type of hybrid system, switched systems can be described as interactions between continuous-time systems and discrete switching events (Liberzon, 2003). It provides a useful and unique paradigm for modelling and control of a large variety of practical engineering systems, especially those subject to complex and stringent dynamics, such as automobile transmission, hybrid powertrain, flight and air traffic systems, power converters, and robotic manipulators, etc. In the controls community, considerable efforts have been devoted over the past decades to the research of switched systems, leading to fruitful results in the literature (see Deaecto et al., 2011; Z. Sun & Ge, 2005; Yuan & Wu, 2015; L. Zhang et al., 2015; Zhao & Hill, 2008 and the references cited therein). In particular, the issues of stability and $\mathcal{L}_2$-gain performance analysis, as well as $\mathcal{H}_\infty$ control design for switched systems were jointly addressed in Zhao and Hill (2008). Deaecto et al. (2011) proposed a novel scheme for the synthesis of dynamic output-feedback control of switched linear systems under the state-dependent min-switching framework (Geromel et al., 2008) using piecewise Lyapunov functions. Yuan and Wu (2015) developed a hybrid switching impulsive controller structure to convexify switching output-feedback control synthesis under the time-dependent average dwell time (ADT) framework (Hespanha & Morse, 1999) using multiple Lyapunov functions. A new switching control logic that mixes the ideas of state-dependent min-switching and time-dependent ADT switching was proposed in Duan and Wu (2014), and a new concept of persistent dwell-time switching extended from the idea of ADT switching was further proposed in L. Zhang et al. (2015). L. Zhang and Gao (2010) considered asynchronous switching issues in stability analysis and control synthesis for switched systems using Lyapunov-like functions. In order to bridge the gap between theoretical research and practical engineering applications, recent research focus in the field has been shifting to accounting various physical constraints for the design of switched control systems, such as actuator saturations (Ma et al., 2016; Wang & Zhao, 2016), uncertain system dynamics (Li, Tong, et al., 2017), quantisation (Cheng et al., 2018; Sui & Tong, 2016), and time delay (Deaecto et al., 2016; Li, Sun, et al., 2017; Yang & Tong, 2015; Zong et al., 2015).

Particularly, the time delay is one of the most commonly seen nonlinearities in practical control systems due to various reasons, such as imperfect actuation and sensing capabilities. Knowing that time delay might degrade the control performance or even destabilise the overall control system, dedicated research efforts have been witnessed in recent years, focused on developing new methodologies and tools, to address the time-delay effects in switched control system design. Several important works are worth to be mentioned. A delay-dependent method was proposed in X. M. Sun et al. (2006) for stability and $\mathcal{L}_2$-gain analysis of switched delay systems. Hetel et al. (2006) developed a switched digital control scheme for continuous-time switched systems, which is robust against time-varying feedback delays. The exponential stability problem was addressed in W. A. Zhang

CONTACT Chengzhi Yuan cyuan@uri.edu Department of Mechanical, Industrial and Systems Engineering, University of Rhode Island, Kingston, 02881 RI, USA

and Yu (2009) for discrete-time switched time-delay systems based on the ADT framework. The model reference output tracking control problem for switched nonlinear systems was studied in Zhai et al. (2015) by combining the ADT technique and the free weighting matrix methods. New criteria for exponential stability of switched time-varying systems with delays were established in Li, Sun, et al. (2017). Furthermore, input-to-state stability was concerned in X. Wu et al. (2019) for time-varying switched systems with time delays. A novel delay-dependent piecewise Lyapunov function was introduced by M. Zhang et al. (2018) for filtering design of switched fuzzy systems subject to mixed time-varying delay and packet dropout effects. Deaecto et al. (2016) addressed the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control problems for time-varying delay switched linear systems by using a new method of modelling time-varying delays as norm-bounded perturbations combined with Lyapunov–Metzler inequalities. Importantly, the results derived therein are also applicable to sampled-data control. In spite of rich literature, we have noted that virtually all of existing methods for analysis and synthesis of time-delay switched control systems are based on constructing a proper form of Lyapunov functionals. One important issue that might be caused is that the resulting analysis and synthesis conditions are non-convex, which could be very difficult to verify and solve. Specifically, with such Lyapunov functional based methods, existing analysis and synthesis conditions for time-delay switched control systems are largely formulated in terms of bilinear matrix inequalities (BMIs), which are well-known NP-hard and need to be solved by global optimisation techniques that are often of high computational complexity. The problem becomes even more challenging if multiple physical constraints are jointly considered, such as system uncertainty and output-feedback control. It thus calls for the development of new methodologies and tools to overcome such an important issue, which motivates our current work.

In this paper, we seek to address the above issue from the perspective of developing new switching controller structures. Specifically, we consider a class of switched systems that are subject to multiple physical constraints, including structured uncertain system dynamics (which can be described in a linear fractional transformation (LFT) form) and time-varying input delays. A novel hybrid controller is proposed, which consists of three major components: the integral quadratic constraint (IQC) dynamics, the continuous dynamics, and the jump dynamics. The IQC dynamics are developed by leveraging tools from classical robust control theory (e.g. Seiler, 2015; Yuan & Wu, 2017b), which aim to address the effects of time-varying input delays. The continuous dynamics are structured by feeding back not only measurement outputs but also some system's internal signals, which are responsible for coping with the structured LFT system uncertainties. The jump dynamics enforce a jump (update/reset) at every switching time instant for the states of both IQC dynamics and continuous dynamics, which are used to accommodate the discrete switching behaviours induced by the controlled plant. Based on this new hybrid controller structure, robust stability of the overall hybrid closed-loop system is established under the ADT switching framework with multiple Lyapunov functions. The associated hybrid control synthesis conditions are fully characterised as linear matrix inequalities (LMIs), which can be solved efficiently via semi-definite programming (Boyd et al., 2004). An application example on regulation of a nonlinear switched electronic circuit system has been used to demonstrate effectiveness and usefulness of the proposed approach.

The contributions of this paper can be summarised in two aspects: (i) a new methodology of applying dynamic IQCs from classical robust control theory is proposed to deal with the time-varying input delay in switched control system design; and (ii) a novel hybrid controller structure is developed to enable convex formulation of the associated switching control synthesis problem in terms of LMIs, such that the stabilising hybrid controller can be synthesised efficiently. It should be noted that the current work is significantly distinguished from two previous works of Yuan and Wu (2015) and (Kao & Rantzer, 2007) in twofold: (i) the work of Yuan and Wu (2015) did not consider the effects of system uncertainties and time-varying input delays; and (ii) the work of Kao and Rantzer (2007) only addressed the stability analysis issue of time-delay systems but did not address the control synthesis issue. In addition, the results established in this paper cannot be obtained by simply combining the methods from Yuan and Wu (2015) and Kao and Rantzer (2007). One technical difficulty lies in how to derive the associated control synthesis conditions in a convex form, which is successfully addressed in this paper by introducing a novel hybrid controller structure under the dynamic IQC framework.

The rest of the paper is organised as follows. Section 2 will first recall some preliminary results on switched systems and dynamic IQCs, followed by the problem statement. The main results, including the new hybrid controller structure and derivation of the associated convex synthesis conditions, are presented in Section 3. Section 4 utilises an application example to illustrate the design procedure and demonstrate effectiveness of the proposed approach. Finally, conclusions are drawn in Section 5.

2. Preliminary and problem statement

2.1 Preliminaries

Notation: Throughout the paper, $\mathbb{R}$ and $\mathbb{C}$ are used to represent the set of real and complex numbers, respectively. $\mathbb{R}_+$ stands for the set of positive real numbers. $\mathbb{R}^{m\times n}$ ($\mathbb{C}^{m\times n}$) is the set of real (complex) $m \times n$ matrices, and $\mathbb{R}^n$ ($\mathbb{C}^n$) represents the set of real (complex) $n \times 1$ vectors. I_n denotes the $n \times n$ identity matrix, and sometimes without causing any confusions I will be slightly abused to denote an identity matrix with appropriate dimension. $\mathbb{S}^n$ and $\mathbb{S}^n_+$ are used to denote the sets of real symmetric $n \times n$ matrices and positive definite matrices, respectively. A block diagonal matrix with matrices $X_1, X_2, \ldots, X_p$ on its main diagonal is denoted by $\operatorname{diag}\{X_1, X_2, \ldots, X_p\}$. For a series of column vectors $x_1, \ldots, x_n$, $col\{x_1, \ldots, x_n\}$ stands for a column vector by stacking them together. The symbol $\star$ in LMIs is used to denote entries that follow from symmetry. For two integers $k_1 < k_2$, we denote $\mathbf{I}[k_1, k_2] = \{k_1, k_1 + 1, \ldots, k_2\}$. For $s \in \mathbb{C}$, $\bar{s}$ denotes the complex conjugate of s. For a matrix $M \in \mathbb{C}^{m\times n}$, M^{T} denotes its transpose and M^* denotes the complex conjugate transpose. The Hermitian operator $He\{\cdot\}$ is defined as $He\{M\} = M + M^{\mathrm{T}}$ for real matrices. $\mathbb{RL}_\infty$ denotes the set of rational functions with real coefficients that are proper and have no poles on the imaginary axis. $\mathbb{RH}_\infty$ is the subset of functions in $\mathbb{RL}_\infty$ that are analytic in the closed right half of the complex plane. $\mathbb{RL}_\infty^{m\times n}$ and $\mathbb{RH}_\infty^{m\times n}$ denote the sets of $m \times n$ matrices whose elements are in $\mathbb{RL}_\infty$ and $\mathbb{RH}_\infty$, respectively. The para-Hermitian conjugate of $G \in \mathbb{RL}_\infty^{m\times n}$, denoted as $G^\sim$, is defined by $G^\sim(s) := G(-\bar{s})^*$. For $x \in \mathbb{C}^n$, its norm is defined as $\|x\| := (x^*x)^{1/2}$. L_{2+}^n is the space of functions $u : [0,\infty) \to \mathbb{R}^n$ satisfying $\|u\|_2 := (\int_0^\infty u^{\mathrm{T}}(t)u(t)\ \mathrm{d}t)^{1/2} < \infty$. Given $u \in L_{2+}^n$, u_T denotes the truncated function $u_T(t) = u(t)$ for $t \leq T$ and $u_T(t) = 0$ otherwise. The extended space, denoted as L_{2e+}, is the set of functions u such that $u_T \in L_{2+}$ for all $T \geq 0$.

Consider a switched linear system described by

$$\dot{x} = A_\sigma x, \tag{1}$$

where x is the state and σ is the switching signal. The system is said to have an average dwell time (ADT) switching logic, if the switching signal σ satisfies the following definition (Hespanha & Morse, 1999).

Definition 2.1: A switching signal σ is said to possess the property of ADT, if there exist two positive numbers N_0 and τ_a such that

$$N_\sigma(t,T) \leq N_0 + \frac{T-t}{\tau_a}, \quad 0 \leq t \leq T, \tag{2}$$

where $N_\sigma(t,T)$ denotes the number of switchings of σ over the time interval (t,T), τ_a and N_0 are called the average dwell time and the chattering bound, respectively.

Some basic definitions and useful lemmas related to dynamic IQCs are recalled as follows.

Definition 2.2 (Seiler, 2015): Let $\Pi \in \mathbb{RL}_\infty^{(m_1+m_2)\times(m_1+m_2)}$ be a proper, rational function, called a 'multiplier', such that $\Pi = \Psi^\sim W \Psi$ with $W \in \mathbb{R}^{n_z\times n_z}$ and $\Psi \in \mathbb{RH}_\infty^{n_z\times(m_1+m_2)}$. Then two signals $q \in L_{2e+}^{m_1}$ and $p \in L_{2e+}^{m_2}$ satisfy the IQC defined by the multiplier Π, and (Ψ, W) is a hard IQC factorisation of Π if the following inequality holds for all $T \geq 0$,

$$\int_0^{\mathrm{T}} z^{\mathrm{T}}(t) W z(t)\ \mathrm{d}t \geq 0, \tag{3}$$

where $z \in \mathbb{R}^{n_z}$ denotes the filtered output of Ψ driven by inputs (q,p) with zero initial conditions, i.e. $z = \Psi \left[\begin{smallmatrix} q \\ p \end{smallmatrix}\right]$. Moreover, a bounded, causal operator $\mathcal{S} : L_{2e+}^{m_1} \to L_{2e+}^{m_2}$ satisfies the IQC defined by Π if condition (3) holds for all $q \in L_{2e+}^{m_1}$, $p = \mathcal{S}(q)$ and all $T \geq 0$.

Note that the factorisation of IQC multiplier $\Pi = \Psi^\sim W \Psi$ is not unique but can be computed with state-space methods (Seiler, 2015). Furthermore, it has been demonstrated that a broad class of IQC multipliers possess a hard factorisation (Megretski & Rantzer, 1997). More discussions about the hard IQCs as defined above can be found in Seiler (2015).

Definition 2.3 (Seiler, 2015): (Ψ, W) is called a J_{m_1,m_2}-spectral factorisation of $\Pi = \Pi^\sim \in \mathbb{RL}_\infty^{(m_1+m_2)\times(m_1+m_2)}$ if $\Pi = \Psi^\sim W \Psi$, $W = \left[\begin{smallmatrix} I_{m_1} & 0 \\ 0 & -I_{m_2} \end{smallmatrix}\right]$, and $\Psi, \Psi^{-1} \in \mathbb{RH}_\infty^{(m_1+m_2)\times(m_1+m_2)}$.

Note that with a J_{m_1,m_2}-spectral factorisation (Ψ, W), Ψ is always square, stable and minimum phase (Seiler, 2015).

2.2 Problem statement

In this paper, we consider the following switched system subject to structured LFT uncertainty and time-varying input delay:

$$\mathcal{P}_\sigma : \begin{cases} \dot{x} = A^\sigma x + B_0^\sigma p + B_2^\sigma \mathcal{D}(u), \\ q = C_0^\sigma x + D_{00}^\sigma p + D_{02}^\sigma \mathcal{D}(u), \\ y = C_2^\sigma x + D_{20}^\sigma p, \\ p = \Delta^\sigma q, \end{cases} \tag{4}$$

where $x \in \mathbb{R}^{n_x}$ is the plant state, $u \in \mathbb{R}^{n_u}$ is the control input, $y \in \mathbb{R}^{n_y}$ is the measurement output, and $p, q \in \mathbb{R}^{n_q}$ are two system's internal signals connecting the structured uncertainty block Δ^σ. The superscript σ denotes a piecewise constant function of time, representing the switching signal, which takes its values in the finite set $\mathbf{I}[1, N_p]$ with $N_p > 1$ denoting the number of subsystems. In this paper, the switching signal σ is assumed to be continuous from the right everywhere and obeys an ADT switching logic as defined in Definition 2.1. The symbol $\mathcal{D}(u)$ denotes a time-varying delay operator defined as $\mathcal{D}(u(t)) := u(t-\tau(t))$, where $\tau(t)$ specifies the time delay amount at the time instant t. We assume that such a time delay amount $\tau(t)$ and its variation are both bounded, i.e. $\tau \in [0, \bar{\tau}]$ and $\dot{\tau} \leq r$ for some positive numbers $\bar{\tau}$ and r. All the associated system matrices are assumed to be known constant matrices of compatible dimensions, in particular (A^i, B_2^i) is stabilisable and (A^i, C_2^i) is detectable for all $i \in \mathbf{I}[1, N_p]$. In addition, the structured uncertainty Δ^i is assumed to be time-varying, norm-bounded, and satisfying the following structure for all $i \in \mathbf{I}[1, N_p]$, i.e.

$$\begin{aligned} \Delta^i \in \Delta = \{ & \mathrm{diag}\{\delta_1 I_{h_1}, \ldots, \delta_s I_{h_s}, \Delta_{s+1}, \ldots, \Delta_{s+f}\} : \\ & \delta_j \in \mathbb{R}, |\delta_j| \leq 1,\ \forall j \in \mathbf{I}[1,s]; \\ & \Delta_{s+k} \in \mathbb{R}^{r_k\times r_k}, \|\Delta_{s+k}\| \leq 1,\ \forall k \in \mathbf{I}[1,f]\}, \end{aligned} \tag{5}$$

where $\sum_{j=1}^s h_j + \sum_{k=1}^f r_k = n_q$, s and f are two positive integers representing the number of scalar sub-blocks δ_j and matrix sub-blocks Δ_{s+k}, respectively. To facilitate the subsequent development, associated with the structured uncertainty (5), we will introduce the following scaling matrix set:

$$\begin{aligned} \Lambda = \{ & \mathrm{diag}\{\Lambda_1, \ldots, \Lambda_s, \lambda_{s+1} I_{r_1}, \ldots, \lambda_{s+f} I_{r_f}\} : \\ & \Lambda_j \in \mathbb{S}_+^{h_j},\ \forall j \in \mathbf{I}[1,s]; \\ & \lambda_{s+k} \in \mathbb{R}_+,\ \forall k \in \mathbf{I}[1,f]\}. \end{aligned} \tag{6}$$

It is observed that for any $\Lambda^i \in \Lambda$, it is commutable with the associated $\Delta^i \in \Delta$, i.e. $\Lambda^i \Delta^i = \Delta^i \Lambda^i$.

Remark 2.1: For those readers that are not familiar with the concept of LFT, we emphasise that the LFT representation adopted above has been widely used in the robust control theory to model various practical engineering systems subject to structured modelling uncertainties, such as magnetic bearing systems (Lauridsen et al., 2015) and electrical circuits (Yuan, 2017a), etc.

In this paper, our objectives are to (i) design a hybrid control law that will stabilise the switched LFT plant (4); and (ii) formulate the associated control synthesis conditions in terms of LMIs, such that the associated control solution can be synthesised efficiently via LMI-based techniques.

3. Main results

3.1 System transformation and hybrid controller structure

Before proposing the controller structure to fulfil the above first objective, it is necessary to reformulate the switched LFT plant dynamics (4) into a new form with two LFT loops. Specifically, by defining a new signal $w(t) = \mathcal{S}(u(t)) := \mathcal{D}(u(t)) - u(t)$ and incorporating it to system (4), it yields

$$\begin{cases} \dot{x} = A^\sigma x + B_0^\sigma p + B_2^\sigma w + B_2^\sigma u, \\ q = C_0^\sigma x + D_{00}^\sigma p + D_{02}^\sigma w + D_{02}^\sigma u, \\ y = C_2^\sigma x + D_{20}^\sigma p, \\ p = \Delta^\sigma q, \\ w = \mathcal{S}(u). \end{cases} \tag{7}$$

This transformed system thus has two LFT loops: the first LFT loop through the structured uncertainty Δ^σ, which is inherited from (4); and the second loop through the new delay difference operator $\mathcal{S}(u)$, which is obtained after performing the above input-delay transformation. It will be clarified in the sequel that such a new LFT reformulation will allow us to tackle the time-varying input delay effects under the dynamic IQC framework as introduced in Section 2.1. To see this, we first introduce the following assumption regarding the delay difference operator $\mathcal{S}(\cdot)$.

Assumption 3.1: *$\mathcal{S}(\cdot)$ satisfies an IQC defined by $\Pi \in \mathbb{RL}_\infty^{2n_u \times 2n_u}$, where the multiplier Π can be partitioned as $\begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12}^\sim & \Pi_{22} \end{bmatrix}$ with Π_{11} of dimension $n_u \times n_u$. Π satisfies $\Pi_{11}(j\omega) > 0$ and $\Pi_{22}(j\omega) < 0$ for all $\omega \in \mathbb{R} \cup \{\infty\}$. Furthermore, Π has a J_{n_u,n_u}-spectral factorisation (Ψ, W) in the form of $\Psi = \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ 0 & I \end{bmatrix} \in \mathbb{RH}_\infty^{(n_u+n_u)\times(n_u+n_u)}$ and $W = \begin{bmatrix} I_{n_u} & 0 \\ 0 & -I_{n_u} \end{bmatrix}$.*

We stress that the above assumption does not cause any loss of generality. The IQC defined therein is used to bound the input-output behaviour of the delay difference operator $\mathcal{S}(\cdot)$. According to Seiler (2015), the strict definiteness assumptions on Π_{11} and Π_{22} are typically adopted in the literature. They guarantee existence of a J_{n_u,n_u}-spectral factorisation (Ψ, W) for the multiplier Π, such that Ψ is square, stable and minimum phase. As such, the last part of this assumption is not restrictive and is made in order to simplify the derivations. In fact, this assumption can be relaxed with more extensive and complicated formulas, we refer interested readers to Yuan and Wu (2017b) for more detailed discussions. Consequently, according to Section 2.1, the above assumption renders an IQC-induced stable LTI system Ψ for the delay difference operator $S(u)$, which can be described in the following state-space form:

$$\text{IQC dynamics: } \begin{cases} \dot{x}_\psi = A_\psi x_\psi + B_{\psi 1} u + B_{\psi 2} w, \\ z = C_\psi x_\psi + D_{\psi 1} u + D_{\psi 2} w, \end{cases} \tag{8}$$

where $x_\psi \in \mathbb{R}^{n_\psi}$ is the state of Ψ, $z \in \mathbb{R}^{2n_u}$ denotes the system outputs, and the associated output matrices satisfy

$$C_\psi = \begin{bmatrix} \bar{C}_\psi \\ 0 \end{bmatrix}, \quad D_{\psi 1} = \begin{bmatrix} \bar{D}_{\psi 1} \\ 0 \end{bmatrix}, \quad D_{\psi 2} = \begin{bmatrix} \bar{D}_{\psi 2} \\ I_{n_u} \end{bmatrix}. \tag{9}$$

Then, based on the transformed switched LFT system (7) and the IQC dynamics (8), we propose to construct the following robust hybrid switching impulsive controller:

Continuous dynamics:

$$\begin{cases} \dot{x}_c = A_{c1}^\sigma x_c + A_{c2}^\sigma x_\psi + B_{c1}^\sigma y + B_{c2}^\sigma p + B_{c3}^\sigma w, \\ u = C_{c1}^\sigma x_c + C_{c2}^\sigma x_\psi + D_{c1}^\sigma y + D_{c2}^\sigma p + D_{c3}^\sigma w, \end{cases} \tag{10}$$

Jump dynamics:

$$\begin{cases} x_\psi^+ = J_{c11}^{ij} x_\psi + J_{c12}^{ij} x_c, \\ x_c^+ = J_{c21}^{ij} x_\psi + J_{c22}^{ij} x_c, \end{cases} \quad \text{when switching occurs,} \tag{11}$$

where $x_c \in \mathbb{R}^{n_c}$ is the controller state with order n_c to be determined, σ is the switching signal from the controlled plant (4). All the associated controller gain matrices are constant matrices to be synthesised, such that the overall closed-loop hybrid system will be stable in the presence of structured uncertainty and time-varying input delays. The two subscripts of the jump dynamics matrices $\begin{bmatrix} J_{c11}^{ij} & J_{c12}^{ij} \\ J_{c21}^{ij} & J_{c22}^{ij} \end{bmatrix}$, i.e. $i, j \in \mathbf{I}[1, N_p]$ with $i \neq j$, are used to denote the indices of the pre-switching subsystem i and the post-switching subsystem j. In other words, at the switching instant when switching occurs, we have $\sigma = i$ and $\sigma^+ = j$. Note that for simplicity of presentation, we have slightly abused throughout this paper the notation $(\cdot)^+$ to denote the value of $(\cdot)$ at time t_s^+ for some switching instant t_s when the switching behaviour occurs.

It is observed that the overall structure of the proposed controller consists of the IQC dynamics (8), the continuous dynamics (10) and the jump dynamics (11). To better understand this new controller structure, we have used Figure 1 to illustrate the interconnecting relationship among these three dynamics components. It can be seen that the continuous dynamics (10) feeds back four signals, including the delay difference signal $w = \mathcal{D}(u) - u$, the controlled plant's internal signal p, and the IQC dynamics state x_ψ, in addition to the measurement output y. The IQC dynamics needs to utilise real-time information of the control input u and the delay difference signal w, while the jump dynamics resets/updates the states of both continuous dynamics and the IQC dynamics at every switching time instant.

Regarding the controller implementation, three important points are needed to be clarified. First, the exact-memory

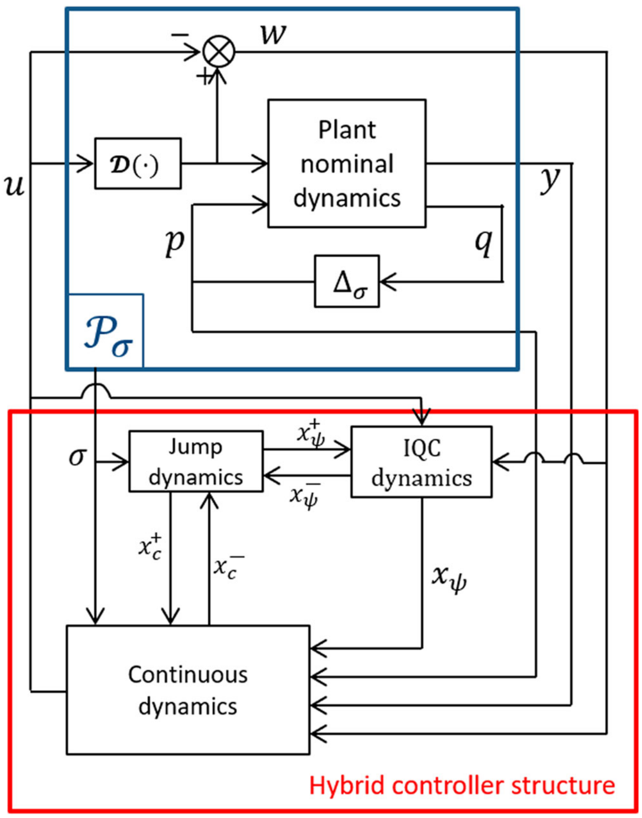

Figure 1. The proposed hybrid control architecture. Blue box includes the controlled plant's switched dynamics; red box includes the proposed IQC-based hybrid controller dynamics.

scheme or the anti-delay scheme from Yuan and Wu (2017a, 2017b), respectively, can be adopted to generate the delay difference signal w. As a result, the IQC dynamics state x_ψ can be readily computed online given the information of u and w. Second, the idea of feeding back the internal signal p for controller implementation is inspired from F. Wu and Lu (2004), whose feasibility has been demonstrated in many practical control systems (e.g. Dai et al., 2009; Yuan, 2017b). This point will be further demonstrated in Section 4 of this paper using a real engineering application example. Third, the continuous dynamics contains an algebraic loop due to the delay difference term $D_{c3}^\sigma w$. To guarantee implementability, this algebraic loop is required to be well-posed, i.e. the matrix $I + D_{c3}^i$ needs to be non-singular for all $i \in \mathbf{I}[1, N_p]$. Once $I + D_{c3}^i$ is non-singular, one can implement the second equation of (10) by

$$\begin{aligned} &\text{If } \tau(t) = 0, \quad u = C_{c1}^\sigma x_c + C_{c2}^\sigma x_\psi + D_{c1}^\sigma y + D_{c2}^\sigma p; \\ &\text{Otherwise,} \quad u = (I + D_{c3}^\sigma)^{-1}[C_{c1}^\sigma x_c + C_{c2}^\sigma x_\psi + D_{c1}^\sigma y \\ &\qquad\qquad + D_{c2}^\sigma p + D_{c3}^\sigma \mathcal{D}(u)]. \end{aligned}$$

This non-singularity issue of $I + D_{c3}^i$ will be addressed in the subsequent development by using the LMI-based control synthesis technique.

The overall closed-loop system resulted from interconnecting the hybrid controller (10)–(11) to the controlled plant (4) and augmenting the IQC dynamics (8) can be obtained as follows by defining $x_{cl} := \mathrm{col}\{x, x_\psi, x_c\}$, $p_{cl} := \mathrm{col}\{p, w\}$, $q_{cl} := \mathrm{col}\{q, z\}$:

$$\text{Continuous dynamics: } \begin{cases} \dot{x}_{cl} = \mathcal{A}^\sigma x_{cl} + \mathcal{B}_0^\sigma p_{cl}, \\ q_{cl} = \mathcal{C}_0^\sigma x_{cl} + \mathcal{D}_{00}^\sigma p_{cl}, \\ p_{cl} = \begin{bmatrix} \Delta^\sigma q \\ \mathcal{S}(u) \end{bmatrix}, \end{cases} \tag{12}$$

$$\text{Jump dynamics: } x_{cl}^+ = \mathcal{J}^{ij} x_{cl}, \quad \text{when switching occurs,} \tag{13}$$

where the closed-loop system matrices are given by

$$\mathcal{A}^\sigma = \begin{bmatrix} \begin{bmatrix} A^\sigma & 0 \\ 0 & A_\psi \end{bmatrix} + \begin{bmatrix} B_2^\sigma \\ B_{\psi 1} \end{bmatrix} [D_{c1}^\sigma \quad C_{c2}^\sigma] \begin{bmatrix} C_2^\sigma & 0 \\ 0 & I \end{bmatrix} & \begin{bmatrix} B_2^\sigma \\ B_{\psi 1} \end{bmatrix} C_{c1}^\sigma \\ [B_{c1}^\sigma \quad A_{c2}^\sigma] \begin{bmatrix} C_2^\sigma & 0 \\ 0 & I \end{bmatrix} & A_{c1}^\sigma \end{bmatrix},$$

$$\mathcal{B}_0^\sigma = \begin{bmatrix} \begin{bmatrix} B_0^\sigma \\ 0 \end{bmatrix} + \begin{bmatrix} B_2^\sigma \\ B_{\psi 1} \end{bmatrix} (D_{c2}^\sigma + D_{c1}^\sigma D_{20}^\sigma) & \begin{bmatrix} B_2^\sigma \\ B_{\psi 2} \end{bmatrix} + \begin{bmatrix} B_2^\sigma \\ B_{\psi 1} \end{bmatrix} D_{c3}^\sigma \\ B_{c2}^\sigma + B_{c1}^\sigma D_{20}^\sigma & B_{c3}^\sigma \end{bmatrix},$$

$$\begin{aligned} \mathcal{C}_0^\sigma &= \begin{bmatrix} \bar{\mathcal{C}}_0^\sigma \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} [C_0^\sigma \quad 0] + D_{02}^\sigma [D_{c1}^\sigma \quad C_{c2}^\sigma] \begin{bmatrix} C_2^\sigma & 0 \\ 0 & I \end{bmatrix} & D_{02}^\sigma C_{c1}^\sigma \\ [0 \quad \bar{C}_\psi] + \bar{D}_{\psi 1} [D_{c1}^\sigma \quad C_{c2}^\sigma] \begin{bmatrix} C_2^\sigma & 0 \\ 0 & I \end{bmatrix} & \bar{D}_{\psi 1} C_{c1}^\sigma \\ 0 & 0 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} \mathcal{D}_{00}^\sigma &= \begin{bmatrix} \bar{\mathcal{D}}_{00}^\sigma \\ [0 \quad I] \end{bmatrix} \\ &= \begin{bmatrix} D_{00}^\sigma + D_{02}^\sigma (D_{c2}^\sigma + D_{c1}^\sigma D_{20}^\sigma) & D_{02}^\sigma + D_{02}^\sigma D_{c3}^\sigma \\ \bar{D}_{\psi 1} (D_{c2}^\sigma + D_{c1}^\sigma D_{20}^\sigma) & \bar{D}_{\psi 2} + \bar{D}_{\psi 1} D_{c3}^\sigma \\ 0 & I \end{bmatrix}, \end{aligned}$$

$$\mathcal{J}^{ij} = \begin{bmatrix} I & 0 & 0 \\ 0 & J_{c11}^{ij} & J_{c12}^{ij} \\ 0 & J_{c21}^{ij} & J_{c22}^{ij} \end{bmatrix}. \tag{14}$$

3.2 Exponential stability analysis and hybrid control synthesis conditions

Based on the new hybrid controller structure proposed in the last section, this section will present the associated control synthesis conditions in terms of LMIs. To this end, the following lemma will be first established to provide analysis conditions that guarantee exponential stability for the hybrid closed-loop system (12)–(13).

Lemma 3.2: *Consider the hybrid system* (12)–(13) *with unstructured uncertainty and time-varying input delays. With Assumption* 3.1 *and given two positive constants* $\lambda_0 \in \mathbb{R}_+$ *and* $\mu > 1$, *if there exist positive definite matrices* $P^i \in \mathbb{S}_+^{n_x+n_\psi+n_c}$, $\Lambda^i \in \Lambda$, *and a positive scalar* $\lambda \in \mathbb{R}_+$, *such that the following conditions* *hold for all* $i, j \in \mathbf{I}[1, N_p]$ *and* $i \neq j$:

$$\begin{bmatrix} He\{P^i\mathcal{A}^i\} + \lambda_0 P^i & \star & \star \\ \mathcal{B}_0^{i\ T} P^i & -\begin{bmatrix} \Lambda^i & 0 \\ 0 & \lambda I \end{bmatrix} & \star \\ \bar{\mathcal{C}}_0^i & \bar{\mathcal{D}}_{00}^i & -\begin{bmatrix} \Lambda^i & 0 \\ 0 & \lambda I \end{bmatrix}^{-1} \end{bmatrix} < 0, \tag{15}$$

$$\begin{bmatrix} \mu P^i & \star \\ P^j \mathcal{J}^{ij} & P^j \end{bmatrix} \geq 0. \tag{16}$$

Then, the hybrid system (12)–(13) *is exponentially stable for every switching signal* σ *with average dwell time* $\tau_a > \frac{\ln(\mu)}{\lambda_0}$.

Proof: Consider the hybrid system (12)–(13), we define the following piecewise Lyapunov function candidate: $V(t) = V^\sigma(x_{cl}) = x_{cl}^{\mathrm{T}} P^\sigma x_{cl}$, where P^σ is switched among the solution P^i's of (15)–(16) in accordance with the piecewise constant switching signal σ. Then, the derivative of each $V^i = x_{cl}^{\mathrm{T}} P^i x_{cl}$ along the solutions of its corresponding subsystem's continuous dynamics satisfies

$$\begin{aligned} \dot{V}^i &= \dot{x}_{cl}^{\mathrm{T}} P^i x_{cl} + x_{cl}^{\mathrm{T}} P^i \dot{x}_{cl} \\ &= (\mathcal{A}^i x_{cl} + \mathcal{B}_0^i p_{cl})^{\mathrm{T}} P^i x_{cl} + x_{cl}^{\mathrm{T}} P^i (\mathcal{A}^i x_{cl} + \mathcal{B}_0^i p_{cl}) \\ &= x_{cl}^{\mathrm{T}} (P^i \mathcal{A}^i + \mathcal{A}^{i\ \mathrm{T}} P^i) x_{cl} + 2 x_{cl}^{\mathrm{T}} P^i \mathcal{B}_0^i p_{cl}. \end{aligned} \tag{17}$$

By Schur complement, condition (15) is equivalent to

$$\begin{bmatrix} He\{P^i\mathcal{A}^i\} + \lambda_0 P^i & \star \\ \mathcal{B}_0^{i\ \mathrm{T}} P^i & -\begin{bmatrix} \Lambda^i & 0 \\ 0 & \lambda I \end{bmatrix} \end{bmatrix} + \begin{bmatrix} \bar{\mathcal{C}}_0^{iT} \\ \bar{\mathcal{D}}_{00}^{iT} \end{bmatrix} \begin{bmatrix} \Lambda^i & 0 \\ 0 & \lambda I \end{bmatrix} [\bar{\mathcal{C}}_0^i \quad \bar{\mathcal{D}}_{00}^i] < 0.$$

Multiplying $[x_{cl}^{\mathrm{T}} \ \ p_{cl}^{\mathrm{T}}]$ from the left-hand side and its transpose from the right of the above matrix inequality yields

$$\begin{aligned} & x_{cl}^{\mathrm{T}} (P^i \mathcal{A}^i + \mathcal{A}^{iT} P^i + \lambda_0 P^i) x_{cl} + 2 x_{cl}^{\mathrm{T}} P^i \mathcal{B}_0^i p_{cl} \\ & \quad + q^{\mathrm{T}} \Lambda^i q - p^{\mathrm{T}} \Lambda^i p + \lambda z^{\mathrm{T}} W z < 0, \end{aligned}$$

where we have utilised the facts of (14) with (9), and that $W = \left[\begin{smallmatrix} I & 0 \\ 0 & -I \end{smallmatrix}\right]$ from Assumption 3.1. This together with (17) implies that

$$\dot{V}^i + \lambda_0 V^i + q^{\mathrm{T}} \Lambda^i q - p^{\mathrm{T}} \Lambda^i p + \lambda z^{\mathrm{T}} W z < 0 \quad \forall i \in \mathbf{I}[1, N_p]. \tag{18}$$

On the other hand, performing Schur complement on condition (16) gives $\mu P^i - \mathcal{J}^{ij\ \mathrm{T}} P^j \mathcal{J}^{ij} \geq 0$, which implies that

$$V^j(x_{cl}^+) \leq \mu V^i(x_{cl}), \quad \forall x_{cl} \in \mathbb{R}^{n_x + n_\psi + n_c} \text{ and } i, j \in \mathbf{I}[1, N_p]. \tag{19}$$

Now, we just need to prove that the conclusion of Lemma 3.2 holds if conditions (18) and (19) are both satisfied. To this end, we first note by substituting $p = \Delta^\sigma q$ into (18) that $q^{\mathrm{T}} \Lambda^i q - p^{\mathrm{T}} \Lambda^i p = q^{\mathrm{T}} \Lambda^i q - q^{\mathrm{T}} \Delta^{i\ \mathrm{T}} \Lambda^i \Delta^i q = q \Lambda^{i^{1/2}} (I - \Delta^{i\ \mathrm{T}} \Delta^i) \Lambda^{i^{1/2}} q \geq 0$ since Δ^i is norm bounded by 1 and Λ^i is commutable with Δ^i. This reduces the result of (18) to

$$\dot{V}^i + \lambda_0 V^i + \lambda z^{\mathrm{T}} W z < 0 \quad \forall i \in \mathbf{I}[1, N_p]. \tag{20}$$

For any given $t > 0$, we let $t_1 < \cdots < t_i$ $(i > 1)$ denote the switching instants of σ over the time interval $(0, t)$. Then, the above inequality implies for all $i \in \mathbf{I}[1, N_p]$ and $t \in [t_i, t_{i+1})$,

$$V^i(t) < V^i(t_i^+) - \int_{t_i}^t \lambda_0 V^i - \int_{t_i}^t \lambda z^{\mathrm{T}} W z \leq V^i(t_i^+) - \int_{t_i}^t \lambda_0 V^i,$$

where the second inequality is obtained by using the fact from (3) that $\int_0^t z^{\mathrm{T}}(\tau) W z(\tau)\, \mathrm{d}\tau \geq 0$ for all $t \geq 0$ under Assumption 3.1, together with $\lambda > 0$. Further, using the differential inequality theory, we obtain from (20) and (19) by induction that

$$\begin{aligned} V(t) &< V(t_i^+)\, \mathrm{e}^{-\lambda_0(t - t_i)} \leq \mu V(t_i)\, \mathrm{e}^{-\lambda_0(t - t_i)} \\ &< \mu V(t_{i-1}^+)\, \mathrm{e}^{-\lambda_0(t_i - t_{i-1})}\, \mathrm{e}^{-\lambda_0(t - t_i)} \leq \cdots \leq \mu^i \mathrm{e}^{-\lambda_0 t} V(0) \\ &= \mu^{N_\sigma(0,t)}\, \mathrm{e}^{-\lambda_0 t} V(0) = \mathrm{e}^{-\lambda_0 t + N_\sigma(0,t) \ln(\mu)} V(0), \end{aligned} \tag{21}$$

where $N_\sigma(\tau, t)$ is the number of switches of σ over the time interval (τ, t). Consequently, with the ADT constraint $\tau_a > \frac{\ln(\mu)}{\lambda_0}$ and (2), the above inequality guarantees $V(t) < \mathrm{e}^{-\lambda_0 t + \frac{\ln(\mu)}{\tau_a} t} V(0) = \mathrm{e}^{(\frac{\ln(\mu)}{\tau_a} - \lambda_0) t} V(0)$ with $\frac{\ln(\mu)}{\tau_a} - \lambda_0 < 0$, implying exponential stability of the overall system, which ends the proof. ■

Remark 3.1: We stress that as opposed to many existing literatures (e.g.Deaecto et al., 2016; X. Wu et al., 2019; M. Zhang et al., 2018) on stability analysis of hybrid impulsive switched systems subject to uncertainties and time delays, Lemma 3.2 provides novel analysis conditions under the dynamic IQC framework combined with multiple Lyapunov functions and the average dwell time technique.

Based on Lemma 3.2, the following theorem presents the synthesis conditions for controller (10)–(11) in terms of LMIs.

Theorem 3.3: *Consider the switched input-delayed LFT uncertain plant (4). With Assumption 3.1 and given two positive constants* $\lambda_0 \in \mathbb{R}_+$ *and* $\mu > 1$, *if there exist positive definite matrices* $R^i \in \mathbb{S}_+^{n_x + n_\psi}$, $S_1^i \in \mathbb{S}_+^{n_x}$, $\hat{\Lambda}^i \in \Lambda$, *rectangular matrices* $S_2^i \in \mathbb{R}^{n_x \times n_\psi}$, $\hat{A}_{c1}^i \in \mathbb{R}^{n_x \times n_\psi}$, $\hat{A}_{c2}^i \in \mathbb{R}^{n_x \times n_x}$, $\hat{B}_{c1}^i \in \mathbb{R}^{n_x \times n_y}$, $\hat{B}_{c2}^i \in \mathbb{R}^{n_x \times n_q}$, $\hat{B}_{c3}^i \in \mathbb{R}^{n_x \times n_u}$, $\hat{C}_{c1}^i \in \mathbb{R}^{n_u \times n_\psi}$, $\hat{C}_{c2}^i \in \mathbb{R}^{n_u \times n_x}$, $\hat{D}_{c1}^i \in \mathbb{R}^{n_u \times n_y}$, $\hat{D}_{c2}^i \in \mathbb{R}^{n_u \times n_q}$, $\hat{D}_{c3}^i \in \mathbb{R}^{n_u \times n_u}$, $\hat{J}_{c11}^{ij} \in \mathbb{R}^{n_\psi \times n_\psi}$, $\hat{J}_{c12}^{ij} \in \mathbb{R}^{n_\psi \times n_x}$, $\hat{J}_{c21}^{ij} \in \mathbb{R}^{n_x \times n_\psi}$, $\hat{J}_{c22}^{ij} \in \mathbb{R}^{n_x \times n_x}$, *and a positive scalar* $\hat{\lambda} \in \mathbb{R}_+$, *such that the following conditions hold for all* $i, j \in \mathbf{I}[1, N_p]$

with $i \neq j$:

$$\begin{bmatrix} He\left\{\begin{bmatrix} A^i & 0 \\ 0 & A_\psi \end{bmatrix} R^i + \begin{bmatrix} B_2^i \\ B_{\psi 1} \end{bmatrix} [\hat{C}_{c2}^i \quad \hat{C}_{c1}^i]\right\} + \lambda_0 R^i & \star & \star & \star & \star & \star \\ [\hat{A}_{c2}^i \quad \hat{A}_{c1}^i] + [A^{i\,T} + \lambda_0 I \quad 0] + C_2^{i\,T} \hat{D}_{c1}^{iT} \begin{bmatrix} B_2^i \\ B_{\psi 1} \end{bmatrix}^T & He\{S_1^i A^i + \hat{B}_{c1}^i C_2^i\} + \lambda_0 S_1^i & \star & \star & \star & \star \\ \hat{\Lambda}^i [B_0^{i\,T} \quad 0] + \hat{D}_{c2}^{iT} [B_2^{i\,T} \quad B_{\psi 1}^T] & \hat{B}_{c2}^{iT} & -\hat{\Lambda}^i & \star & \star & \star \\ \hat{\lambda} [B_2^{i\,T} \quad B_{\psi 2}^T] + \hat{D}_{c3}^{iT} [B_2^{i\,T} \quad B_{\psi 1}^T] & \hat{B}_{c3}^{iT} & 0 & -\hat{\lambda} I & \star & \star \\ [C_0^i \quad 0] R^i + D_{02}^i [\hat{C}_{c2}^i \quad \hat{C}_{c1}^i] & C_0^i + D_{02}^i \hat{D}_{c1}^i C_2^i & D_{00}^i \hat{\Lambda}^i + D_{02}^i \hat{D}_{c2}^i & \hat{\lambda} D_{02}^i + D_{02}^i \hat{D}_{c3}^i & -\hat{\Lambda}^i & \star \\ [0 \quad \bar{C}_\psi] R^i + \bar{D}_{\psi 1} [\hat{C}_{c2}^i \quad \hat{C}_{c1}^i] & \bar{D}_{\psi 1} \hat{D}_{c1}^i C_2^i & \bar{D}_{\psi 1} \hat{D}_{c2}^i & \hat{\lambda} \bar{D}_{\psi 2} + \bar{D}_{\psi 1} \hat{D}_{c3}^i & 0 & -\hat{\lambda} I \end{bmatrix} < 0, \tag{22}$$

$$\begin{bmatrix} \mu R^i & \star & \star & \star \\ \mu [I \quad 0] & \mu S_1^i & \star & \star \\ \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} R^i + \begin{bmatrix} 0 & 0 \\ \hat{J}_{c12}^{ij} & \hat{J}_{c11}^{ij} \end{bmatrix} & \begin{bmatrix} I \\ 0 \end{bmatrix} & R^j & \star \\ [\hat{J}_{c22}^{ij} \quad \hat{J}_{c21}^{ij}] & S_1^j & [I \quad 0] & S_1^j \end{bmatrix} \geq 0,$$

$$\begin{bmatrix} R^i & \star \\ [I \quad 0] & S_1^i \end{bmatrix} > 0, \tag{23}$$

$$\begin{bmatrix} \hat{\lambda} I & \star \\ \hat{D}_{c3}^i & \hat{\lambda} I \end{bmatrix} > 0. \tag{24}$$

Then, the hybrid controller (10)–(11) with IQC dynamics (8) renders the overall closed-loop system exponentially stable for every switching signal σ with average dwell time $\tau_a > \frac{\ln(\mu)}{\lambda_0}$. Moreover, the controller (10)–(11) of order $n_c = n_x$ is well-posed, and its coefficient matrices can be reconstructed through the following algorithm:

- *For all $i \in \mathbf{I}[1, N_p]$, partition matrices $R^i = \begin{bmatrix} R_1^i & R_2^i \\ R_2^{i\,T} & R_3^i \end{bmatrix}$, $R^{i\,-1} = \begin{bmatrix} \grave{R}_1^i & \grave{R}_2^i \\ \grave{R}_2^{iT} & \grave{R}_3^i \end{bmatrix}$ with $R_1^i, \grave{R}_1^i \in \mathbb{S}_+^{n_x}$, $R_2^i, \grave{R}_2^i \in \mathbb{R}^{n_x \times n_\psi}$, $R_3^i, \grave{R}_3^i \in \mathbb{S}_+^{n_\psi}$ and let $S_3^i = \grave{R}_3^i + (\grave{R}_2^i - S_2^i)^T (S_1^i - \grave{R}_1^i)^{-1} (\grave{R}_2^i - S_2^i)$. Then we have $S^i := \begin{bmatrix} S_1^i & S_2^i \\ S_2^{i\,T} & S_3^i \end{bmatrix} > 0$. Note that the matrix $S_1^i - \grave{R}_1^i$ is guaranteed to be invertible, according to Yuan and Wu (2015).*
- *For all $i \in \mathbf{I}[1, N_p]$, solve $N^i \in \mathbb{R}^{(n_x + n_\psi) \times n_x}$ through the factorisation $S^i - R^{i\,-1} = N^i Q^i N^{i\,T}$, where $Q^i \in \mathbb{S}_+^{n_x}$, and define $M^i := -R^i N^i Q^i$ so that $S^i R^i + N^i M^{i\,T} = I$. Furthermore, we partition M^i, N^i as $M^i = [M_1^{i\,T} \quad M_2^{i\,T}]^T$, $N^i = [N_1^{i\,T} \quad N_2^{i\,T}]^T$ so that $M_1^i, N_1^i \in \mathbb{R}^{n_x \times n_x}$ are invertible and $M_2^i, N_2^i \in \mathbb{R}^{n_\psi \times n_x}$.*
- *For all $i, j \in \mathbf{I}[1, N_p]$ with $i \neq j$, compute the controller gain matrices via*

$$\begin{aligned}
&D_{c1}^i = \hat{D}_{c1}^i, \quad D_{c2}^i = \hat{D}_{c2}^i (\hat{\Lambda}^i)^{-1} - D_{c1}^i D_{20}^i, \\
&D_{c3}^i = \hat{\lambda}^{-1} \hat{D}_{c3}^i, \\
&B_{c1}^i = N_1^{i\,-1} \left(\hat{B}_{c1}^i - [S_1^i \quad S_2^i] \begin{bmatrix} B_2^i \\ B_{\psi 1} \end{bmatrix} D_{c1}^i \right), \\
&B_{c2}^i = N_1^{i\,-1} \left(\hat{B}_{c2}^i - [S_1^i \quad S_2^i] \begin{bmatrix} B_2^i \\ B_{\psi 1} \end{bmatrix} D_{c2}^i \right. \\
&\qquad \left. - [S_1^i \quad S_2^i] \begin{bmatrix} B_0^i \\ 0 \end{bmatrix} \hat{\Lambda}^i \right) (\hat{\Lambda}^i)^{-1} - B_{c1}^i D_{20}^i, \\
&B_{c3}^i = (\hat{\lambda} N_1^i)^{-1} \left(\hat{B}_{c3}^i - [S_1^i \quad S_2^i] \right. \\
&\qquad \left. - \left(\hat{\lambda} \begin{bmatrix} B_2^i \\ B_{\psi 2} \end{bmatrix} - \begin{bmatrix} B_2^i \\ B_{\psi 1} \end{bmatrix} \hat{D}_{c3}^i \right) \right), \\
&[C_{c2}^i \quad C_{c1}^i] = ([\hat{C}_{c2}^i \quad \hat{C}_{c1}^i] - D_{c1}^i [C_2^i \quad 0] R^i) \Omega^{i\,-1}, \\
&[A_{c2}^i \quad A_{c1}^i] = N_1^{i\,-1} \left\{ [\hat{A}_{c2}^i \quad \hat{A}_{c1}^i] - [S_1^i \quad S_2^i] \right. \\
&\qquad \left(\begin{bmatrix} A^i & 0 \\ 0 & A_\psi \end{bmatrix} R^i + \begin{bmatrix} B_2^i \\ B_{\psi 1} \end{bmatrix} [\hat{C}_{c2}^i \quad \hat{C}_{c1}^i] \right) \\
&\qquad \left. - N_1^i B_{c1}^i [C_2^i \quad 0] R^i \right\} \Omega^{i\,-1}, \\
&[J_{c11}^{ij} \quad J_{c12}^{ij}] = [\hat{J}_{c12}^{ij} \quad \hat{J}_{c11}^{ij}] \Omega^{i\,-1}, \\
&[J_{c21}^{ij} \quad J_{c22}^{ij}] = N_1^{j\,-1} \{ [\hat{J}_{c22}^{ij} \quad \hat{J}_{c21}^{ij}] \\
&\qquad - [S_1^j \quad 0] R^i - S_2^j [\hat{J}_{c12}^{ij} \quad \hat{J}_{c11}^{ij}] \} \Omega^{i\,-1},
\end{aligned} \tag{25}$$

where $\Omega^i := \begin{bmatrix} R_2^{i\,T} & R_3^i \\ M_1^{i\,T} & M_2^{i\,T} \end{bmatrix}$.

Proof: From Lemma 3.2, we have proved that exponential stability of the hybrid closed-loop system (12)–(13) can be guaranteed under conditions (15)–(16). As such, in order to transform these two analysis conditions into convex control synthesis conditions, we specify the associated Lyapunov function matrices as

$$P^i = \begin{bmatrix} S^i & N^i \\ N^{i\,T} & X^i \end{bmatrix} = \left[\begin{array}{cc|c} S_1^i & S_2^i & N_1^i \\ S_2^{i\,T} & S_3^i & N_2^i \\ N_1^{i\,T} & N_2^{i\,T} & X^i \end{array}\right]$$

and let

$$Z_1^i = \left[\begin{array}{c|c} R^i & \begin{matrix} I \\ 0 \end{matrix} \\ M^{i\,T} & 0 \end{array}\right] = \left[\begin{array}{cc|c} R_1^i & R_2^i & I \\ R_2^{i\,T} & R_3^i & 0 \\ M_1^{i\,T} & M_2^{i\,T} & 0 \end{array}\right],$$

$$Z_2^i = \begin{bmatrix} I & 0 & S_1^i \\ 0 & I & S_2^{i\,\mathrm{T}} \\ 0 & 0 & N_1^{i\,\mathrm{T}} \end{bmatrix},$$

such that $P^iZ_1^i = Z_2^i$ and $N^iM^{i\,\mathrm{T}} = I - S^iR^i$, which gives $X^iM^{i\,\mathrm{T}} = -N^{i\,\mathrm{T}}R^i$. Then, we perform the congruent transformations on conditions (15) and (16) with matrices diag $\left\{Z_1^i, \left[\begin{smallmatrix} \hat{\Lambda}^i & 0 \\ 0 & \hat{\lambda} \end{smallmatrix}\right], I, I, I\right\}$ and diag$\{Z_1^i, Z_1^j\}$, respectively, where $\hat{\Lambda}^i := \Lambda^{i\,-1}$ and $\hat{\lambda} := \lambda^{-1}$. This yields the following results:

$$Z_1^{i\,\mathrm{T}}P^iZ_1^i = Z_2^{i\,\mathrm{T}}Z_1^i = \begin{bmatrix} R^i & \begin{bmatrix} I \\ 0 \end{bmatrix} \\ [I \quad 0] & S_1^i \end{bmatrix},$$

$$Z_1^{i\,\mathrm{T}}P^i\mathcal{A}^iZ_1^i = Z_2^{i\,\mathrm{T}}$$

$$\mathcal{A}^iZ_1^i = \begin{bmatrix} \begin{bmatrix} A^i & 0 \\ 0 & A_\psi \end{bmatrix} R^i + \begin{bmatrix} B_2^i \\ B_{\psi 1} \end{bmatrix} [\hat{C}_{c2}^i \quad \hat{C}_{c1}^i] & \begin{bmatrix} A^i \\ 0 \end{bmatrix} + \begin{bmatrix} B_2^i \\ B_{\psi 1} \end{bmatrix} \hat{D}_{c1}^i C_2^i \\ [\hat{A}_{c2}^i \quad \hat{A}_{c1}^i] & S_1^iA^i + \hat{B}_{c1}^iC_2^i \end{bmatrix},$$

$$\begin{bmatrix} \hat{\Lambda}^i & 0 \\ 0 & \hat{\lambda}I \end{bmatrix} \mathcal{B}_0^{i\,\mathrm{T}}P^iZ_1^i = \begin{bmatrix} \hat{\Lambda}^i & 0 \\ 0 & \hat{\lambda}I \end{bmatrix} \mathcal{B}_0^{i\,\mathrm{T}}$$

$$Z_2^i = \left[\hat{\Lambda}^i[B_0^{i\,\mathrm{T}} \quad 0] + \hat{D}_{c2}^{iT}[B_2^{i\,\mathrm{T}} \quad B_{\psi 1}^{\mathrm{T}}] \quad \hat{\lambda}[B_2^{i\,\mathrm{T}} \quad B_{\psi 2}^{\mathrm{T}}] + \hat{D}_{c3}^{iT}[B_2^{i\,\mathrm{T}} \quad B_{\psi 1}^{\mathrm{T}}] \quad \hat{B}_{c3}^{iT}\right],$$

$$\bar{\mathcal{C}}_0^iZ_1^i = \begin{bmatrix} [C_0^i \quad 0]R^i + D_{02}^i[\hat{C}_{c2}^i \quad \hat{C}_{c1}^i] & C_0^i + D_{02}^i\hat{D}_{c1}^iC_2^i \\ [0 \quad \bar{C}_\psi]R^i + \bar{D}_{\psi 1}[\hat{C}_{c2}^i \quad \hat{C}_{c1}^i] & \bar{D}_{\psi 1}\hat{D}_{c1}^iC_2^i \end{bmatrix},$$

$$\bar{\mathcal{D}}_{00}^i \begin{bmatrix} \hat{\Lambda}^i & 0 \\ 0 & \hat{\lambda}I \end{bmatrix} = \begin{bmatrix} D_{00}^i\hat{\Lambda}^i + D_{02}^i\hat{D}_{c2}^i & \hat{\lambda}D_{02}^i + D_{02}^i\hat{D}_{c3}^i \\ \bar{D}_{\psi 1}\hat{D}_{c2}^i & \hat{\lambda}\bar{D}_{\psi 2} + \bar{D}_{\psi 1}\hat{D}_{c3}^i \end{bmatrix},$$

$$Z_1^{j\,\mathrm{T}}P^j\mathcal{J}^{ij}Z_1^i = Z_2^{j\,\mathrm{T}}$$

$$\mathcal{J}^{ij}Z_1^i = \begin{bmatrix} \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix} R^i + \begin{bmatrix} 0 & 0 \\ \hat{J}_{c12}^{ij} & \hat{J}_{c11}^{ij} \end{bmatrix} & \begin{bmatrix} I \\ 0 \end{bmatrix} \\ [\hat{J}_{c22}^{ij} \quad \hat{J}_{c21}^{ij}] & S_1^j \end{bmatrix},$$

where

$$\hat{D}_{c1}^i = D_{c1}^i, \quad \hat{D}_{c2}^i = (D_{c2}^i + D_{c1}^iD_{20}^i)\hat{\Lambda}^i, \quad \hat{D}_{c3}^i = \hat{\lambda}D_{c3}^i,$$

$$\hat{B}_{c1}^i = [S_1^i \quad S_2^i]\begin{bmatrix} B_2^i \\ B_{\psi 1} \end{bmatrix} D_{c1}^i + N_1^iB_{c1}^i,$$

$$\hat{B}_{c2}^i = [S_1^i \quad S_2^i]\left(\begin{bmatrix} B_0^i \\ 0 \end{bmatrix}\hat{\Lambda}^i + \begin{bmatrix} B_2^i \\ B_{\psi 1} \end{bmatrix}\hat{D}_{c2}^i\right) + N_1^i(B_{c2}^i + B_{c1}^iD_{20}^i)\hat{\Lambda}^i,$$

$$\hat{B}_{c3}^i = [S_1^i \quad S_2^i]\left(\hat{\lambda}\begin{bmatrix} B_2^i \\ B_{\psi 2} \end{bmatrix} + \begin{bmatrix} B_2^i \\ B_{\psi 1} \end{bmatrix}\hat{D}_{c3}^i\right) + \hat{\lambda}N_1^iB_{c3}^i,$$

$$[\hat{C}_{c2}^i \quad \hat{C}_{c1}^i] = D_{c1}^i[C_2^i \quad 0]R^i + [C_{c2}^i \quad C_{c1}^i]\Omega^i,$$

$$[\hat{A}_{c2}^i \quad \hat{A}_{c1}^i] = [S_1^i \quad S_2^i]\left(\begin{bmatrix} A^i & 0 \\ 0 & A_\psi \end{bmatrix}R^i + \begin{bmatrix} B_2^i \\ B_{\psi 1} \end{bmatrix}[\hat{C}_{c2}^i \quad \hat{C}_{c1}^i]\right) + N_1^iB_{c1}^i[C_2^i \quad 0]R^i + N_1^i[A_{c2}^i \quad A_{c1}^i]\Omega^i,$$

$$[\hat{J}_{c12}^{ij} \quad \hat{J}_{c11}^{ij}] = [J_{c11}^{ij} \quad J_{c12}^{ij}]\Omega^i,$$

$$[\hat{J}_{c22}^{ij} \quad \hat{J}_{c21}^{ij}] = [S_1^j \quad 0]R^i + S_2^j[\hat{J}_{c12}^{ij} \quad \hat{J}_{c11}^{ij}] + N_1^j[J_{c21}^{ij} \quad J_{c22}^{ij}]\Omega^i. \tag{26}$$

Consequently, conditions (15) and (16) are transformed equivalently to conditions (22) and (23), respectively. In particular, the second part of condition (23) is used to ensure that the Lyapunov matrix P^i is positive definite. Moreover, condition (24) is used to guarantee the resulting controller in the form of (10)–(11) is well-posed. Specifically, this condition implies via Schur complement that $\hat{D}_{c3}^{iT}\hat{\lambda}^{-1}\hat{D}_{c3}^i - \hat{\lambda}I < 0$, which is equivalent to $D_{c3}^{i\,\mathrm{T}}D_{c3}^i < I$, ensuring invertibility of the matrix $I + D_{c3}^i$, in turn guaranteeing implementability of the controller as discussed in Subsection 3.1. Finally, the controller gain matrices reconstruction formulae (25) can be verified by inverting the relations in (26), which ends the proof. ∎

Remark 3.2: It is observed that the hybrid control synthesis conditions (22)–(24) are fully characterised as LMIs, which can be solved efficiently by semi-definite programming techniques (Boyd et al., 2004). Afterwards, the associated hybrid controller can be reconstructed by using the algorithm provided in Theorem 3.3. The computational complexity of solving the LMI conditions (22)–(24) will be discussed in the simulation section using a specific example.

Remark 3.3: It should be noted that the synthesis conditions established in Theorem 3.3 are convex with a pre-chosen (fixed) IQC dynamics, which could facilitate solving the associated control synthesis problem but at a price of yielding potentially conservative results. How to overcome such a deficiency by considering the IQC dynamics as free variables with balanced computational complexity and synthesis conservatism remains an open problem in the field, which will be investigated in our future work. In addition, instead of employing just one single dynamic IQC for dealing with the time-varying input delays, it is promising to further reduce conservatism by employing conic combinations of multiple dynamic IQCs (e.g.Pfifer & Seiler, 2015a, 2016). The influence of different IQC representations (e.g.Kao, 2012; Pfifer & Seiler, 2015b) is another promising direction worth further investigation for reducing conservatism.

Remark 3.4: Note that the two positive constants λ_0 and μ need to be selected in advance in order to reach a convex hybrid control synthesis. These two constants can be selected by satisfying the average dwell time constraint $\tau_a > \frac{\ln(\mu)}{\lambda_0}$. As indicated in the conditions in both Lemma 2 and Theorem 1, the constant μ is upper bounding the possible increase of the values of each pair of Lyapunov functions at each switching time instant, and the constant λ_0 constrains the convergence rate of each Lyapunov function at the continuous-time intervals when no switching occurs. As such, it is expectable that a larger μ might lead to larger abrupt jumping effects at the switching time instants, and a larger λ_0 might result in faster convergence speed at continuous time intervals, vice versa. Extensive studies on the effects of μ and λ_0 to switched system performance have been given in Lu et al. (2006), which are not repeated in this paper. We refer interested readers to these references for more details.

4. Application example

In this section, we seek to demonstrate the effectiveness and usefulness of the proposed hybrid control approach by applying it to solve an important engineering problem of regulation of a nonlinear switched electronic circuit system. Specifically, we consider the electronic circuit as shown in Figure 2, whose switched dynamics can be obtained as follows based on the Kirchhoff's law:

$$\begin{cases} C_{b\sigma}\dfrac{\mathrm{d}v_b}{\mathrm{d}t} = \dfrac{v_a - v_b}{R_\sigma} - f(v_b) - u, \\ C_{a\sigma}\dfrac{\mathrm{d}v_a}{\mathrm{d}t} = \dfrac{v_b - v_a}{R_\sigma} - i_L, \\ L\dfrac{\mathrm{d}i_L}{\mathrm{d}t} = v_a, \end{cases} \tag{27}$$

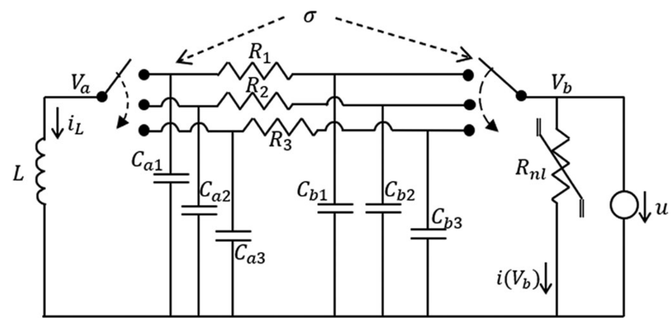

Figure 2. Nonlinear switched electronic circuit.

where σ is the switching signal with $\sigma \in \{1, 2, 3\}$, v_a, v_b are two voltage signals crossing over the resistor R_σ, i_L is the current flowing through the inductor L, and u represents an input current signal. $C_{a\sigma}$ and $C_{b\sigma}$ with $\sigma \in \{1, 2, 3\}$ are capacitors. The characteristic of the nonlinear resistor $f(v_b)$ is a cubic function (Yuan, 2017b) defined by $f(v_b) = av_b + cv_b^3$ with $a < 0$, $c > 0$ being two constant numbers.

The regulation control problem for the above switched electronic circuit aims to ensure stability of the overall system and drive the voltage output v_a to zero in the presence of the switching event σ and the time-varying input delays. To formulate the problem under the proposed design framework, we let $x = [v_b, v_a, i_L]^{\mathrm{T}}$, and define uncertain parameters $\Delta^\sigma = \left[\begin{smallmatrix}\delta^\sigma & 0 \\ 0 & \delta^\sigma\end{smallmatrix}\right]$ with $\delta^\sigma := \frac{v_b}{\bar{v}^\sigma}$ being normalised such that $\Delta^i \in \Delta$ (for $i = 1$, 2, 3), where we assume that $v_b \in [-\bar{v}^\sigma, \bar{v}^\sigma]$ with $\bar{v}^i > 0$ being two positive constants. As a result, the original dynamics of the switched electronic circuit (27) can be equivalently rewritten in the following switched LFT form:

$$\begin{cases} \dot{x} = \begin{bmatrix} -\left(\dfrac{a}{C_b^\sigma} + \dfrac{1}{R^\sigma C_b^\sigma}\right) & \dfrac{1}{R^\sigma C_b^\sigma} & 0 \\ \dfrac{1}{R^\sigma C_a^\sigma} & -\dfrac{1}{R^\sigma C_a^\sigma} & -\dfrac{1}{C_a^\sigma} \\ 0 & \dfrac{1}{L} & 0 \end{bmatrix} \\ \qquad x + \begin{bmatrix} 0 & -\dfrac{c}{C_b^\sigma} \\ 0 & 0 \\ 0 & 0 \end{bmatrix} p + \begin{bmatrix} -\dfrac{1}{C_b^\sigma} \\ 0 \\ 0 \end{bmatrix} \mathcal{D}(u), \\ q = \begin{bmatrix} \bar{v}^\sigma & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ \bar{v}^\sigma & 0 \end{bmatrix} p, \\ y = [1 \quad 0 \quad 0]x, \\ p = \Delta^\sigma q. \end{cases} \tag{28}$$

Note that we have assumed that the voltage v_b is measurable for feedback control. As such, since p in (28) is related to the system state by $p = [v_b^2, v_b^3]^{\mathrm{T}}$, p is also measurable for feedback control use, which ensures implementability of the continuous dynamics of our proposed robust hybrid control strategy. Moreover, we have also assumed that the control input u is subject to time-varying delays with $\mathcal{D}(u) = u(t - \tau(t))$, where $\tau(t)$ satisfies $\tau \in [0, \bar{\tau}]$ and $\dot{\tau} \leq r$. For IQC-based delay control synthesis, we specify the two constant parameters $\bar{\tau} = 0.1$ and $r = 0.01$. Then, we can select the following dynamic IQC multiplier Π

from Kao and Rantzer (2007) to characterise the associated time-delay difference operator $\mathcal{S}(u)$ as defined in (7), i.e.

$$\Pi(s) = \begin{bmatrix} |\varphi(s)|^2 & 0 \\ 0 & -1 \end{bmatrix},$$

where $\varphi(s) = k(\frac{\bar{\tau}^2 s^2 + c'\bar{\tau}s}{\bar{\tau}^2 s^2 + a'\bar{\tau}s + b'}) + \delta'$, with $k = \sqrt{\frac{8}{2-r}}$, $a' = \sqrt{6.5 + 2b'}$, $b' = \sqrt{50}$, $c' = \sqrt{12.5}$, and δ' is an arbitrarily small positive number selected as $\delta' = 0.0001$. By applying the IQC factorisation methods of Seiler (2015), one can obtain the J-spectral factorisation of the above Π as

$$\Psi(s) = \begin{bmatrix} \left(\dfrac{\frac{k(c'-a')}{\bar{\tau}} s - kb'/\bar{\tau}^2}{s^2 + \frac{a'}{\bar{\tau}} s + b'/\bar{\tau}^2} + k + \delta' \right) & 0 \\ 0 & 1 \end{bmatrix}. \tag{29}$$

It is easily verified that such a resulting IQC-induced LTI system Ψ satisfies Assumption 3.1. Moreover, the associated IQC-induced dynamics can be expressed in a state-space form of (8) with the system matrices are given by

$$A_\psi = \begin{bmatrix} 0 & 1 \\ -\dfrac{b'}{\bar{\tau}^2} & -\dfrac{a'}{\bar{\tau}} \end{bmatrix}, \quad B_{\psi 1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_{\psi 2} = 0,$$

$$\bar{C}_\psi = \begin{bmatrix} -\dfrac{b'k}{\bar{\tau}^2} & \dfrac{k(c'-a')}{\bar{\tau}} \end{bmatrix}, \quad \bar{D}_{\psi 1} = k + \delta', \quad \bar{D}_{\psi 2} = 0.$$

Based on the above system setup, we chose the following system parameters with normalised values to solve the LMI conditions (22)–(24): $R_1 = 10/7$, $R_2 = 1$, $R_3 = 1.5$, $C_{b1} = 1/9$, $C_{b2} = 3$, $C_{b3} = 2$, $C_{a1} = 2$, $C_{a2} = 1$, $C_{a3} = 1.5$, $L = 1/7$, $\bar{v}^1 = \bar{v}^2 = 5$, $a = -0.1$, $c = 2/45$ and $\mu = 2$, $\lambda_0 = 0.15$. The resulting controller gain matrices can be further obtained through (25). To show the computational efficiency in solving the above convex LMIs, we adopt the LMI solver in MATLAB on an Intel Core i7-8750H CPU with 32GB RAM to conduct the control synthesis process, which involves 13 LMIs and 74 matrix variables while only takes 0.089 seconds.

Finally, with initial conditions $x(0) = [3, 2, 1]^{\mathrm{T}}$, $x_\psi(0) = [0, 0]^{\mathrm{T}}$, $x_c(0) = [0, 0, 0]^{\mathrm{T}}$, and a time-varying input delay profile $\tau(t) = 0.99 + 0.01\sin(t)$, we carry out the time-domain simulation on the original nonlinear switched electronic circuit by using the synthesised hybrid controller. The simulation results are plotted in Figures 3–7. In particular, Figure 3 shows the switching signal $\sigma(t)$, indicating that a total of 8 switches occur throughout the entire 50 sec simulation. This implies an average dwell time $\tau_a = \frac{50}{8} = 6.25$, which is larger than $\frac{\ln(\mu)}{\lambda_0} = \frac{\ln(2)}{0.15} = 4.6210$, satisfying the associated average dwell time condition in Theorem 3.3. All the system states, including the plant states, the IQC dynamics states, and the controller states, are displayed in Figures 4–6, respectively, which demonstrates stability of the overall system. The control input signal u is also included in Figure 7 to illustrate feasibility of the proposed approach. To further demonstrate advantage of the proposed approach, we compare the simulation results with those obtained by the conventional robust control approach of Zhou et al. (1996) using a common Lyapunov function for the switched system. It turns out that the conventional robust control approach is unable

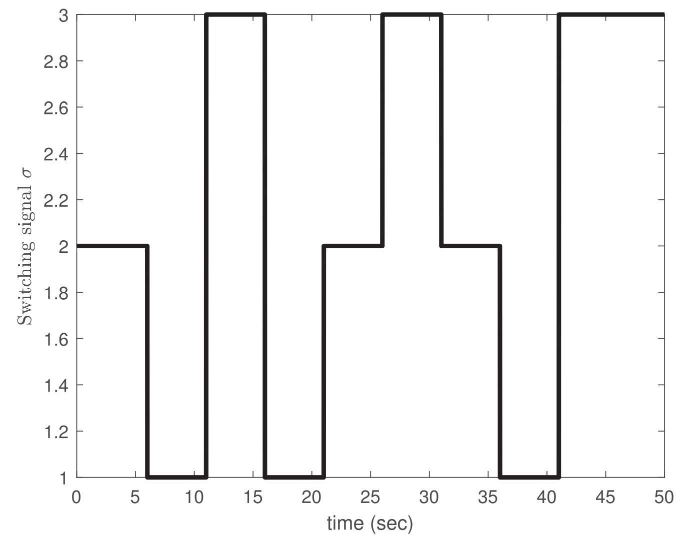

Figure 3. Switching signal σ.

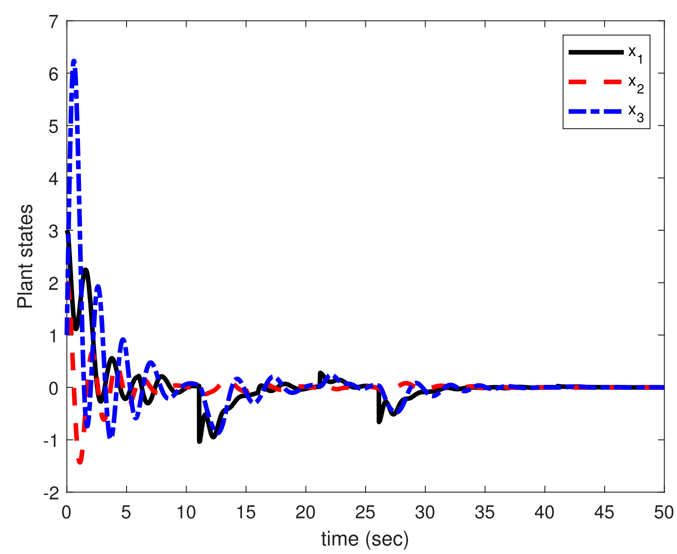

Figure 4. Plant states x.

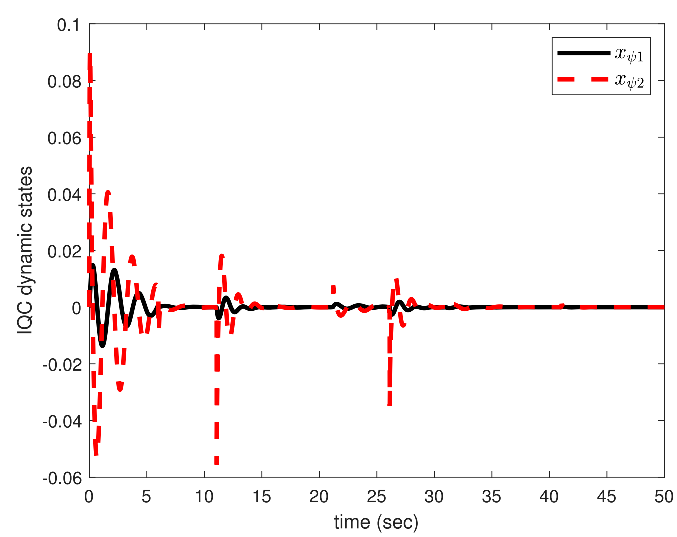

Figure 5. IQC dynamics states x_ψ.

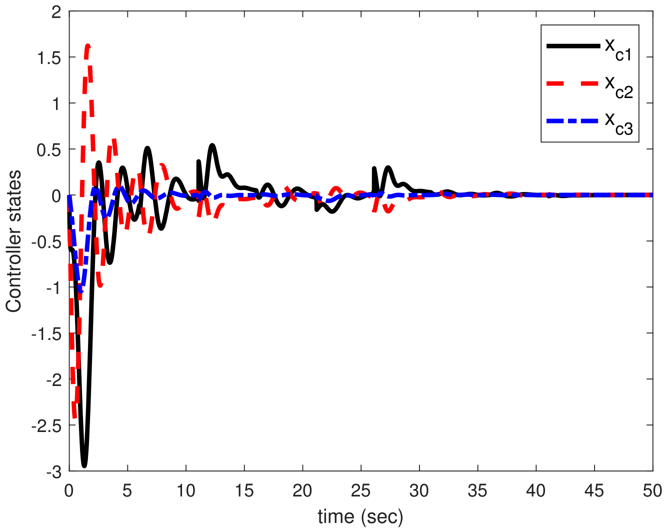

Figure 6. Controller states x_c.

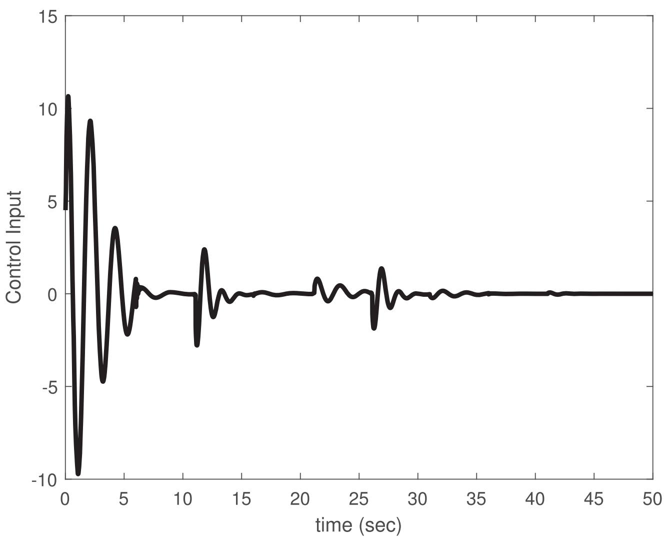

Figure 7. Control input u.

to yield a feasible solution for this application example. Note that advantages of the IQC-based method over existing Lyapunov functional methods for control of time-delay systems have been extensively studied in Yuan and Wu (2017b), which is not repeated here.

5. Conclusions

In this paper, we have proposed a novel hybrid controller for robust control of a class of switched uncertain systems subject to structured LFT uncertainties and time-varying input delays. The proposed hybrid controller is compelling in the sense that (i) it employs dynamic IQCs to cope with the effects of time-varying delays; (ii) it utilises not only measurement outputs but also some system's internal signals for feedback control; (iii) it contains a jump dynamics to enforce controller state jump at each switching time instant; and more importantly (iv) the associated control synthesis conditions that guarantee exponential stability can be fully characterised as LMIs under the ADT switching framework, which can be solved efficiently. Finally, the proposed approach has been successfully applied to solve the regulation control problem for a nonlinear switched electronic circuit system. For future work, it is promising to extend the proposed methodology to hybrid systems with more general settings, such as switched systems with nonlinear unstructured uncertainties, switched impulsive systems, and hybrid systems with state delays.

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