Switching Model Predictive Control of Switched Linear Systems with Average Dwell Time

Chengzhi Yuan, Yan Gu, Wei Zeng, Paolo Stegagno

***Abstract*— In this paper, we address the switching model predictive control (sMPC) problem for a class of switched linear systems with average dwell time (ADT) switching logics. A novel state-feedback switching control synthesis scheme is proposed, such that (i) the sMPC design, subject to ADT switching as well as input and output constraints, can be characterized as an optimization problem of the "worst-case" objective function over infinite moving horizon; (ii) the associated optimal switching control synthesis conditions can be fully formulated as linear matrix inequalities (LMIs), which can be solved efficiently via online convex optimization; and (iii) asymptotic stability of the resulting switched closed-loop system can be proved rigorously using multiple Lyapunov functions. A numerical example has been used to demonstrate effectiveness of the proposed approach.**

I. Introduction

As an important class of hybrid systems, switched systems are composed by a family of continuous-time subsystems and a discrete-time switching logic governing the switching behavior among them [1]. Switched systems can be used to model a large variety of practical engineering systems, such as automobile transmission systems, hybrid powertrain systems, electrical converting circuits, digital control systems, and robotic manipulators, etc. The study of switched systems has experienced a long history involving diverse disciplines. In the controls community, control design of switched systems is one of most thriving research topics over the past decades, leading to fruitful interesting results that can be found in the literature (see, e.g., [2], [3], [4], [5], [6], [7], [8] and the references cited therein).

Among many different control problems for switched systems, optimal switching control represents the most challenging one that has not been well addressed to date. This is owing to the existence of the switching logics that have to be taken into account for optimal switching control designs, making many existing powerful tools tailored to traditional non-switched control systems not applicable to switched systems. Existing methodologies for dealing with the switching logics in switched control systems include min/max switching [9], hysteresis switching [10], average dwell time (ADT) switching [11], and their hybridizations [12]. In particular, the ADT switching method has been recognized very flexible in switched system analysis and switching control synthesis [1], [13], [14]. This is because of its unique property that allows the system to switch fast if necessary and compensate it by switching slowly later on for ensuring overall switching stability. Based on such different switching logics, various optimal switching control techniques have been developed in recent years. For example, [5] proposed a novel scheme for synthesis of optimal $\mathcal{H}_\infty$ dynamic output-feedback control of switched linear systems under the state-dependent min-switching framework using piecewise Lyapunov functions. [6] developed a hybrid switching impulsive controller structure to convexify optimal $\mathcal{H}_\infty$ switching output-feedback control synthesis under the time-dependent ADT framework using multiple Lyapunov functions. A new switching control logic that mixes the ideas of state-dependent min-switching and time-dependent ADT switching was proposed in [12], and the associated optimal $\mathcal{H}_\infty$ switching control synthesis conditions were fully cast as LMIs that can be solved efficiently via convex optimization. Moreover, a new concept of persistent dwell-time switching extended from the idea of ADT switching was further proposed in [7]. [15], [16] considered asynchronous switching issues in stability analysis and optimal control synthesis for switched systems using Lyapunov-like functions. One common trait of all the above-mentioned works in handling optimal switching control synthesis is that they all rely on off-line optimization techniques using off-line system information only (e.g., system model) but ignoring richer online information (e.g., system state/output measurements). Advantages of off-line optimization are noticeable, including capability of handling complex optimization problems that could demand very high computational power; and stability guarantee of the resulting control systems. However, its also suffers from critical drawbacks, including the lack of adaptability to unexpected system variations (e.g., disturbance effects); and potential conservatism in optimal control synthesis.

Model predictive control (MPC) [17] provides promising techniques to overcome such drawbacks through online (instead of off-line) optimization. The key idea of MPC is to solve an online optimization problem at each step, so as to compute an optimal control profile over finite/infinite horizon

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of future time. At each sampling time step, through online optimization a sequence of predicted control actions will be calculated, but only the first action will be implemented. At the next sampling time step, the optimization problem will be solved again with new measurements, and the control input will be updated accordingly. The problem of MPC for traditional non-switched systems has been extensively studied and has found wide applications in engineering practices (such as industrial/chemical process controls [18], [19], [20], [21]). However, research on MPC for switched systems has gained limited success. Several seminal works in the field are worth being mentioned. [22], [23] addressed the so-called hybrid MPC problem for switched systems with state-dependent switching logics. One limitation of the approaches proposed therein is that they require switching mode variations during the prediction horizon to be known *a priori* for the MPC optimization at each step, which is not always feasible in practice. This limitation was surmounted in [24] for switched linear systems by considering a time-dependent dwell time switching logic. The MPC problem for switched nonlinear systems was also studied in [25] using an ADT switching logic which is more flexible than the case considered in [24]; however, this work was primarily focused on system analysis, the control synthesis issue has yet to be more adequately addressed.

In this paper, we seek to further push the development of the field of MPC for switched systems by proposing a new switching MPC synthesis scheme that would render the associated online optimization problem convex (ensuring that the MPC problem can be solved in a computationally efficient manner) and the resulting switched closed-loop system stable. Specifically, we consider an important class of switched linear systems subject to ADT switching constraints as well as input and output bounded constraints. A new state-feedback switching control synthesis scheme is proposed, the novelties of which can be reflected in the follows aspects: (i) The switching MPC design is characterized as an online optimization problem of the "worst-case" objective function over infinite moving horizon. (ii) The associated optimal switching control synthesis conditions are fully formulated as linear matrix inequalities (LMIs), which can be solved efficiently via online convex optimization [26]. (iii) Asymptotic stability of the resulting switched closed-loop system is rigorously proved using multiple Lyapunov functions. A numerical example has been used to demonstrate effectiveness of the proposed scheme.

The rest of this paper is organized as follows. The problem is formulated in Section II. The main results proposed in this paper are presented in Section III. Section IV illustrates the proposed approach through numerical simulations. The conclusions are drawn in Section V.

The notation used throughout this paper is standard. $\mathbb{R}$ stands for the set of real numbers and $\mathbb{R}_+$ represents the set of positive real numbers. $\mathbb{R}^n$ denotes n-dimensional real vector set, and $\mathbb{R}^{m\times n}$ is the set of real $m\times n$ matrices. The transpose of a real matrix M is denoted by M^T. A block diagonal matrix with matrices $X_1,\cdots,X_p$ on its diagonal will be denoted by $diag\{X_1,\cdots,X_p\}$. $\mathbb{S}^n$ and $\mathbb{S}^n_+$ are used to denote the set of real symmetric $n\times n$ matrices and positive definite matrices, respectively. If $M\in\mathbb{S}^n$, then $M>0$ ($M\geq 0$) indicates that M is positive definite (positive semi-definite), and $M<0$ ($M\leq 0$) means negative definite (negative semi-definite). For $x\in\mathbb{R}^n$, the Euclidean norm is $\|x\|:=(x^Tx)^{\frac{1}{2}}$. An infinite sequence $x:=\{x_1,x_2,\cdots\}$ with $x_i\in\mathbb{R}$, is said to be in ℓ_2 if $\sum_{i=1}^{\infty}x_i^Tx_i<\infty$. Furthermore, we use the symbol $\star$ in LMIs to denote entries that follow from symmetry. For two integers $k_1<k_2$, we denote $\mathbf{I}[k_1,k_2]=\{k_1,k_1+1,\cdots,k_2\}$.

II. Problem Statement

Consider a discrete-time switched linear system, whose dynamics can be described by

$$\begin{aligned} x(k+1) &= A_\sigma x(k)+B_\sigma u(k),\\ y(k) &= C_\sigma x(k)+D_\sigma u(k), \end{aligned} \tag{1}$$

where $x\in\mathbb{R}^n$ is the state, $u\in\mathbb{R}^{n_u}$ is the control input, and $y\in\mathbb{R}^{n_y}$ is the system output. σ is a piecewise constant function of time, called the switching signal, which takes its value in the finite set $\mathbf{I}[1,N_p]$ with $N_p>1$ denoting the number of subsystems. In this paper, we are interested in a class of switching signals that satisfy an average dwell time (ADT) constraint. Specifically, according to [11], a switching signal σ is said to have average dwell time τ_a if there exist numbers $N_0,\tau_a>0$ such that

$$\forall K\geq k\geq 0: \quad N_\sigma(K,k)\leq N_0+\frac{K-k}{\tau_a},$$

where $N_\sigma(K,k)$ is the number of switches occurring in the time interval $[k,K)$. For each $\sigma\in\mathbf{I}[1,N_p]$, $(A_\sigma,B_\sigma,C_\sigma,D_\sigma)$ are constant matrices with compatible dimensions. We assume that the pair (A_σ,B_σ) is stabilizable for all $\sigma\in\mathbf{I}[1,N]$.

Our objective is to synthesize a state-feedback controller for the switched linear system (1) under the ADT switching logic, such that the switching model predictive control (sMPC) problem can be solved with computationally-efficient synthesis conditions.

Similar to the traditional MPC problem (e.g., [19], [27]), the sMPC problem under consideration consists of a step-by-step optimization, where in each step new measurements will be obtained and a pre-specified cost function depending on the predicted future plant states will be minimized. More specific, let $x(k+i|k)$, $y(k+i|k)$ and $u(k+i|k)$ be the predicted state, output, and control action, respectively, for step $k+i$ based on the measurement at step k. Particularly, $i=0$ implies that $x(k|k)=x(k)$, $y(k|k)=y(k)$, and $u(k|k)=u(k)$. The cost function to be minimized at step k is specified as

$$\begin{aligned} J_\infty(k):=\sum_{i=0}^{\infty}\Big\{&x^T(k+i|k)Q_{\sigma(k)}x(k+i|k)\\ &+u^T(k+i|k)R_{\sigma(k)}u(k+i|k)\Big\}, \end{aligned} \tag{2}$$

where $Q_\sigma \in \mathbb{S}^n$ ($Q_\sigma \geq 0$) and $R_\sigma \in \mathbb{S}^{n_u}$ ($R_\sigma \geq 0$) for all $\sigma \in \mathbf{I}[1, N_p]$ are tunable parameters. Then, the sMPC problem can be formally formulated as follows.

Definition 1: Given the switched linear plant (1) with an ADT switching signal σ, and state measurement at sampling time k as $x(k)$, define the p-tuple of integers $\{h_1, \cdots, h_p\}$ such that $\sum_{\xi=1}^{p} h_\xi = n_u$ and the q-tuple of integers $\{m_1, \cdots, m_q\}$ such that $\sum_{\zeta=1}^{q} m_\zeta = n_y$, and partition the input and output conformably. The sMPC problem at step k is feasible if the following constrained optimization problem:

$$\begin{aligned} &\min_{u(k+i|k), i=0,1,\cdots} J_\infty(k) \\ &\text{s.t.} \quad \max_{i\geq 0} \|u_\xi(k+i|k)\| \leq u_{\xi,max}, \quad \forall \xi \in \mathbf{I}[1,p] \\ &\qquad\;\; \max_{i\geq 0} \|y_\zeta(k+i|k)\| \leq y_{\zeta,max}, \quad \forall \zeta \in \mathbf{I}[1,q]. \end{aligned} \tag{3}$$

is solvable for all $\sigma \in \mathbf{I}[1, N_p]$ satisfying the ADT switching constraint.

Note that the sMPC problem considers constraints on both system input and output in terms of component-wise peak bounds and Euclidean norm bounds over infinite time horizon, as well as the switching signals in terms of ADT.

III. Main Results

In this section, to solve the above sMPC problem, a state-feedback switching control strategy with a novel computationally-efficient synthesis procedure based on the LMI framework will be proposed, followed by rigorous analysis on switching stability of the resulting switched closed-loop system.

Specifically, we seek to synthesize a switching state-feedback control law for (1) at each step k in the form of

$$u(k+i|k) = F_\sigma(k) x(k+i|k), \tag{4}$$

which will minimize the cost function (2) for all $i = 0, 1, \cdots$. The resulting switched closed-loop system can be formulated as

$$\begin{aligned} x(k+i+1|k) &= (A_\sigma + B_\sigma F_\sigma) x(k+i|k), \\ y(k+i|k) &= (C_\sigma + D_\sigma F_\sigma) x(k+i|k). \end{aligned} \tag{5}$$

With (5), we have the following theorem to summarize the associated LMI synthesis conditions.

Theorem 1: Consider the switched linear system consisting of the plant (1) and the controller (4). Given constant scalars $u_{\xi,max}$ ($\xi \in \mathbf{I}[1,p]$), $y_{\zeta,max}$ ($\zeta \in \mathbf{I}[1,q]$), and $\mu \geq 1$, $0 < \alpha < 1$, if there exist matrices $X_\sigma \in \mathbb{S}_+^n$, $Y_\sigma \in \mathbb{R}^{n_u \times n}$, and a positive scalar $\gamma > 0$, such that

$$\begin{bmatrix} (1-\alpha)X_{\sigma(k)} & \star & \star & \star \\ A_{\sigma(k)}X_{\sigma(k)} + B_{\sigma(k)}Y_{\sigma(k)} & X_{\sigma(k)} & \star & \star \\ Q_{\sigma(k)}^{1/2} X_{\sigma(k)} & 0 & \gamma & \star \\ R_{\sigma(k)}^{1/2} Y_{\sigma(k)} & 0 & 0 & \gamma \end{bmatrix} > 0, \tag{6}$$

$$\begin{bmatrix} 1 & \star \\ x(k) & X_{\sigma(k)} \end{bmatrix} \geq 0, \tag{7}$$

$$\begin{bmatrix} u_{\xi,max}^2 I & \star \\ Y_{\sigma(k),\xi}^T & X_{\sigma(k)} \end{bmatrix} \geq 0, \tag{8}$$

$$\begin{bmatrix} y_{\zeta,max}^2 X_{\sigma(k)} & \star \\ C_{\sigma(k),\zeta} X_{\sigma(k)} + D_{\sigma(k),\zeta} Y_{\sigma(k)} & I \end{bmatrix} \geq 0, \tag{9}$$

$$\begin{bmatrix} \mu V_{\sigma(k-1)}(x(k-1)) & \star \\ x(k) & X_{\sigma(k)} \end{bmatrix} \geq 0, \quad \text{only when switch occurs at time } k, \text{ i.e., } \sigma(k-1) \neq \sigma(k). \tag{10}$$

where $V_{\sigma(k-1)}(x(k-1)) = x^T(k-1) X_{\sigma(k-1)}^{-1} x(k-1)$, the rows of Y_σ, and the rows of C_σ, D_σ are partitioned conformably to the p-tuple and q-tuple, respectively. Then the sMPC problem as defined in Definition 1 is solvable by a stabilizing switching state-feedback controller (4) with $F_\sigma = Y_\sigma X_\sigma^{-1}$ for any switching signal σ satisfying the ADT $\tau_a > \tau_a^* = -\frac{\ln(\mu)}{\ln(1-\alpha)}$.

Proof: Consider the switched closed-loop system (5) with ADT switching, we follow the ideas from [28] to define a switching Lyapunov function as $V_\sigma(x) := x^T X_\sigma^{-1} x$. Then, from condition (6), we can obtain the following by congruence transformation with matrix $diag\{X_{\sigma(k)}^{-1}, I, I, I\}$ and in light of $F_\sigma = Y_\sigma X_\sigma^{-1}$:

$$\begin{bmatrix} (1-\alpha)X_{\sigma(k)}^{-1} & \star & \star & \star \\ A_{\sigma(k)} + B_{\sigma(k)}F_{\sigma(k)} & X_{\sigma(k)} & \star & \star \\ Q_{\sigma(k)}^{1/2} & 0 & \gamma & \star \\ R_{\sigma(k)}^{1/2} F_{\sigma(k)} & 0 & 0 & \gamma \end{bmatrix} > 0.$$

By Schur complement, we can further obtain

$$\begin{aligned} (1-\alpha)X_{\sigma(k)}^{-1} &- \frac{1}{\gamma}(Q_{\sigma(k)} + F_{\sigma(k)}^T R_{\sigma(k)} F_{\sigma(k)}) \\ &- (A_{\sigma(k)} + B_{\sigma(k)}F_{\sigma(k)})^T X_{\sigma(k)}^{-1} \\ &\times (A_{\sigma(k)} + B_{\sigma(k)}F_{\sigma(k)}) > 0. \end{aligned} \tag{11}$$

Pre-multiplying $x^T(k+i|k)$ and post-multiplying its transpose on both sides of the above equation, and using the facts that $Q_{\sigma(k)}, R_{\sigma(k)} \geq 0$, it yields

$$\begin{aligned} &(1-\alpha)x^T(k+i|k) X_\sigma^{-1} x(k+i|k) \\ &> x^T(k+i+1|k) X_\sigma x(k+i+1|k), \ \forall i \geq 0. \end{aligned} \tag{12}$$

Since $0 < \alpha < 1$, the above inequality implies that when no switching occurs at time step k, the synthesized sequence $\{x^T(k+i|k) X_\sigma^{-1} x(k+i|k)\}_{i=0}^{\infty}$ is exponentially decreasing. We further examine the case when switching occurs at time step k by examining condition (10), which by Schur complement yields

$$\mu x^T(k-1) X_{\sigma(k-1)}^{-1} x(k-1) \geq x^T(k) X_{\sigma(k)}^{-1} x(k). \tag{13}$$

Conditions (12) and (13) can be rewritten as follows in terms of the Lyapunov function, respectively.

$$\begin{aligned} &V_\sigma(x(k+i+1|k)) - V_\sigma(x(k+i|k)) < -\alpha V_\sigma(x(k+i|k)), \\ &V_{\sigma(k)} \le \mu V_{\sigma(k-1)}, \quad \begin{array}{c}\text{when switch occurs at } k, \\ \text{i.e., } \sigma(k) \neq \sigma(k-1)\end{array} \end{aligned} \tag{14}$$

According to [28], [29], condition (14) indicates that the sequence $\{x^T(k+i|k)X_\sigma^{-1}x(k+i|k)\}_{i=0}^{\infty}$ is asymptotically decreasing to 0 for any switching signal σ satisfying the ADT constraint $\tau_a > \tau_a^* = -\frac{\ln(\mu)}{\ln(1-\alpha)}$, thus the switched system is asymptotically stabilized.

We further examine the sMPC performance. Multiplying (11) from the left by $x^T(k+i|k)$ and its transpose from the right side, we can get

$$\begin{aligned} &\frac{1}{\gamma}(x^T(k+i|k)Q_\sigma x(k+i|k) + u^T(k+i|k)R_\sigma u(k+i|k)) \\ &< (1-\alpha)V_\sigma(x(k+i|k)) - V_\sigma(x(k+i+1|k)) \\ &< V_\sigma(x(k+i|k)) - V_\sigma(x(k+i+1|k)), \end{aligned} \tag{15}$$

Summing the above results from $i = 0$ to $i = \infty$, we have

$$\begin{aligned} J_\infty(k) = &\sum_{i=0}^{\infty} \Big\{ x^T(k+i|k)Q_\sigma x(k+i|k) \\ &\quad + u^T(k+i|k)R_\sigma u(k+i|k) \Big\} \\ &< \gamma V_\sigma(x(k)) - V_\sigma(x(k+\infty|k)) \\ &\le \gamma V_\sigma(x(k)) \\ &\le \gamma, \end{aligned} \tag{16}$$

where the last inequality can be obtained from condition (7) which through Schur complement yields $1 - x^T(k)X_\sigma^{-1}x(k) = 1 - V_\sigma(x(k)) \ge 0$.

By congruence transformation, condition (9) is equivalent to

$$\begin{bmatrix} y_{\zeta,max}^2 X_\sigma^{-1} & \star \\ C_{\sigma,\zeta} + D_{\sigma,\zeta}F_\sigma & I \end{bmatrix} \ge 0.$$

Then, for each partitioned output, we have

$$y_{\zeta,max}^2 X_\sigma^{-1} - (C_{\sigma,\zeta} + D_{\sigma,\zeta}F_\sigma)^T(C_{\sigma,\zeta} + D_{\sigma,\zeta}F_\sigma) \ge 0,$$

which together with condition (7) indicates that for any $i \ge 0$,

$$\|y_\zeta(k+i|k)\|^2 = \|(C_{\sigma,\zeta} + D_{\sigma,\zeta}F_\sigma)x(k+i|k)\|^2 \le y_{\zeta,max}^2.$$

Similarly for each partitioned input, we have

$$\begin{aligned} \max_{i\ge 0} \|u_\xi(k+i|k)\|^2 &= \max_{i\ge 0} \|Y_{\sigma,\xi}X_\sigma^{-1}x(k+i|k)\|^2 \\ &\le \max_{v^TX_\sigma^{-1}v\le 1} \|Y_{\sigma,\xi}X_\sigma^{-1}v\|^2 \\ &\le \lambda_{max}(Y_{\sigma,\xi}X_\sigma^{-1}Y_{\sigma,\xi}^T) \\ &\le u_{\xi,max}^2, \end{aligned}$$

where $\lambda_{max}(M)$ denotes the largest eigenvalue of M, the last inequality is obtained under condition (8). This ends the proof. ■

Remark 1: Note that condition (10) needs to be included in online sMPC synthesis only when the system is experiencing a switching. As seen from the proof, this condition allows the associated Lyapunov function to be discontinuous and exhibit certain jumps (with an upper limit) at each switching time instant while still ensuring switching stability under the ADT constraint, the idea of which is inherited from the traditional ADT switching stability theory [1]. Solving such a condition online requires to not only measure the instantaneous plant state information but also memorize one-step-past state information for calculating the value of $V_{\sigma(k-1)}(x(k-1))$.

Remark 2: From the above proof and the definition of (2), the sMPC performance requirement (3) can be optimized by solving the following LMI optimization problem:

$$\begin{aligned} &\min_{X_\sigma, Y_\sigma} \gamma \\ &\text{s.t. } (6)\text{–}(10). \end{aligned} \tag{17}$$

Theorem 1 provides a set of LMI conditions that enable online synthesis of the switching state-feedback controller in a computationally-efficient way, and also ensure switching stability if the same state-feedback controller is implemented after step k. Since the optimization problem will be solved for each time step, the stability of the overall switched closed-loop system over the entire horizon needs to be further examined. To this end, the following theorem is further established.

Theorem 2: The feasible switching state-feedback control law provided by Theorem 1 stabilizes the switched linear system (1) asymptotically for any switching signal satisfying the ADT $\tau_a > \tau_a^* = -\frac{\ln(\mu)}{\ln(1-\alpha)}$.

Proof: For clarity of presentation, the optimal solution of LMI conditions (6)–(10) at time step k will be denoted by $X_{\sigma,k}$, $Y_{\sigma,k}$, etc. Then, we consider the same switching Lyapunov function as defined in the proof of Theorem 1 for the switched closed-loop system (5). From conditions (6) and (7), we can obtain

$$x^T(k+1|k)X_{\sigma,k}^{-1}x(k+1|k) < (1-\alpha)x^T(k)X_{\sigma,k}^{-1}x(k) \le 1. \tag{18}$$

This indicates that when no switching occurs, the optimal solution synthesized at time step k is also a feasible solution of conditions (6)–(9) for all time steps afterwards. In addition, because $X_{\sigma,k+1}$ is the optimal solution but $X_{\sigma,k}$ is only a feasible solution at step $k+1$, it implies that

$$x^T(k+1)X_{\sigma,k+1}^{-1}x(k+1) \le x^T(k+1)X_{\sigma,k}^{-1}x(k+1),$$

which together with (18) yields

$$V_{\sigma,k+1}(x(k+1)) < (1-\alpha)V_{\sigma,k}(x(k)), \tag{19}$$

for all $k \ge 0$ when no switching occurs. When switching occurs, say at time step k, the following results can be obtained from condition (10) by following the same procedure of deriving (13) in the proof of Theorem 1:

$$V_{\sigma,k}(x(k)) \le \mu V_{\sigma,k-1}(x(k-1)). \tag{20}$$

Thus, according to [28], [29], (19) and (20) guarantee that $V_{\sigma(k)}(x(k))$ is exponentially converging to zero for any switching signal σ satisfying the ADT constraint $\tau_a > \tau_a^* = -\frac{\ln(\mu)}{\ln(1-\alpha)}$, and in turns $\lim_{k\to\infty} x(k) = 0$ given that X_σ is positive definite. ■

IV. Example

In this section, a numerical example will be used to illustrate the design procedure and control performance of the proposed sMPC approach. Consider a discrete-time switched linear system in the form of (1) with the system matrices given by

$$A_\sigma = \begin{bmatrix} 1 & 0.2\sigma \\ -0.1\sigma & -\sigma \end{bmatrix}, \quad B_\sigma = \begin{bmatrix} 0 \\ \sigma \end{bmatrix},$$
$$C_\sigma = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D_\sigma = 0, \quad \sigma = 1, 2.$$

We assume that the switched system is subject to the following input and output constraints:

$$\|u(k)\| \leq 1, \quad \|y(k)\| \leq 2.5.$$

For online LMI-based sMPC synthesis, we select the ADT coefficients $\mu = 2$ and $\alpha = 0.05$, which leads to $\tau_a^* = -\frac{\ln(\mu)}{\ln(1-\alpha)} = 13.5134$. With initial conditions $x(0) = [-2, 2.5]$, we implement the proposed sMPC law to obtain the system responses as plotted in Fig. 1. Specifically, it is shown in Fig. 1(a) that the overall switched system is indeed stabilized with both states converging to zero asymptotically despite the existence of switching behaviors. This is guaranteed by the fact that the switching signal (as shown in Fig. 1(c)) satisfies the ADT constraint, i.e., $\tau_a^* < \frac{150}{10} = 15$ where 10 is the number of switches occurred in total over the operation time $[0, 150]$. The control input signal shown in Fig. 1(b) also meets the bounded constraint. Fig. 1(d) shows the real-time profile of γ obtained via online LMI-based optimization.

V. Conclusions

In this paper, a new switching state-feedback control scheme has been proposed to address the problem of model predictive control (MPC) for an important class of switched linear systems with average dwell time (ADT). With this scheme, the MPC design considering both input and output constraints was formulated as an online constrained optimization problem, and the associated control synthesis conditions were derived and fully cast as linear matrix inequalities, which can be solved efficiently online via convex optimization. Rigorous analysis has been conducted to demonstrate stability of the overall switching control system. A numerical example has also been used to show effectiveness of the proposed theoretical results.

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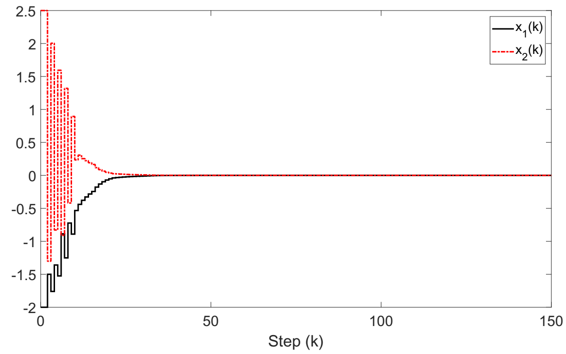
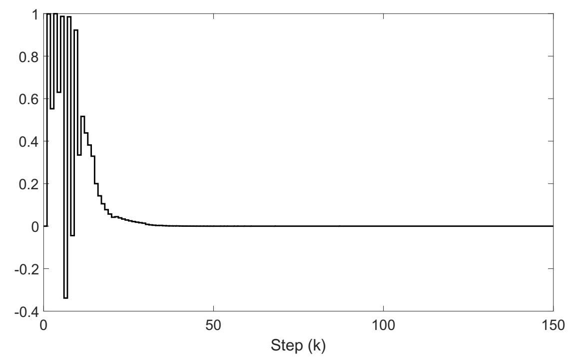
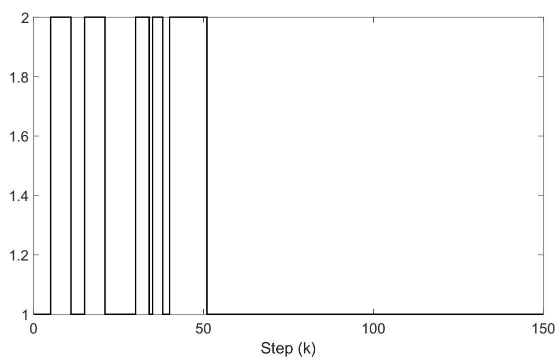


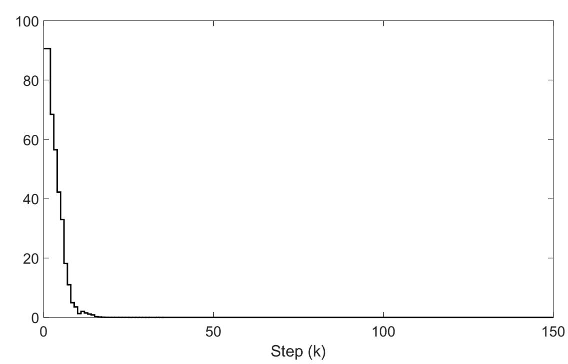
(a) plant states $x(k)$



(b) control input $u(k)$



(c) switching signal $\sigma(k)$



(d) optimized value of $\gamma(k)$

Fig. 1: System responses with the proposed sMPC law.

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