

Supplementary Materials

1 INTRODUCTION

This document contains supplementary materials for the paper with title "Provably Stabilizing Controllers for Quadrupedal Robot Locomotion on Dynamic Rigid Platforms".

2 DESCRIPTION OF FIGURES

2.1 Additional Validation Results with a Periodically Pitching Rigid Platform

Figures 1 and 2 display the validation results obtained from MATLAB, Pybullet, and experiments with the reference gaits (G2) and (G3), respectively, for robot walking on a periodically pitching rigid platform (i.e., the platform (P1)). These figures demonstrate effectiveness of the proposed control approach similar to Fig. 6 in Section VI-C of the main manuscript.

The PD gains for gait (G2) are chosen as $\mathbf{K}_p = \text{diag}(100, 36, 121, 100, 36, 121, 36, 64, 121)$ and $\mathbf{K}_d = \text{diag}(20, 12, 22, 20, 12, 22, 12, 16, 22)$. The PD gains for gait (G3) are chosen as $\mathbf{K}_p = \text{diag}(100, 49, 121, 100, 49, 121, 36, 64, 121)$ and $\mathbf{K}_d = \text{diag}(20, 14, 22, 20, 14, 22, 12, 16, 22)$.

Specifically, Figures 1 (a) and 2 (a) show the joint trajectory tracking results, which demonstrate that reliable trajectory tracking is achieved through both simulations and experiments. Figures 1 (b) and 2 (b) show the base roll and pitch trajectories, which both indicate that the robot keeps a relatively steady base pose while walking on the pitching platform. Figures 1 (c) and 2 (c) show that simulations and experiments share relatively consistent trends in the joint torque profiles.

2.2 Additional Validation Results on Robustness

Figure 3 displays the controller validation results obtained from MATLAB, Pybullet, and experiments with the reference gait (G2) under the uncertainties (U2). The PD gains are chosen the same as the gait (G2).

Interpretation of the results are given in Section VI-D of the main manuscript, which is quoted next:

"With the uncertainties (U2), up to $\pm 20\%$ uncertainties, which approximately correspond to a variation of 8 cm in the stance-foot height over 10 gait cycles, are added to the belt speed of the treadmill. The robot's motion was shaky during experiments, but the robot was able to sustain motion for over twenty steps, which indicates that the inherent robustness of the proposed control approach is able to tackle the implemented uncertainties in the treadmill belt speed."

2.3 Reference Trajectories for Gait (G1)-G(3)

Reference trajectories used for controller validation are shown in Figs. 4-6. As explained in Section VI-A of the main manuscript, these trajectories are generated using the optimization-based planning method introduced in Section V of the main manuscript.

LIST OF FIGURES

LIST OF FIGURES

1	Trajectory tracking results with gait (G2): (a) joint position trajectories, (b) base roll and pitch trajectories, and (c) joint torque profiles of the rear-left leg.	3
2	Trajectory tracking results with gait (G3): (a) joint position trajectories, (b) base roll and pitch trajectories, and (c) joint torque profiles of the rear-left leg.	4
3	Trajectory tracking results with gait (G2) under uncertainties (U2): (a) joint position trajectories, (b) base roll and pitch trajectories, and (c) joint torque profiles of the rear-left leg.	5
4	Reference trajectory for the gait (G1): (a) joint reference trajectories and (b) base roll and pitch reference trajectories.	6
5	Reference trajectory for the gait (G2): (a) joint reference trajectories and (b) base roll and pitch reference trajectories.	7
6	Reference trajectory for the gait (G3): (a) joint reference trajectories and (b) base roll and pitch reference trajectories.	8

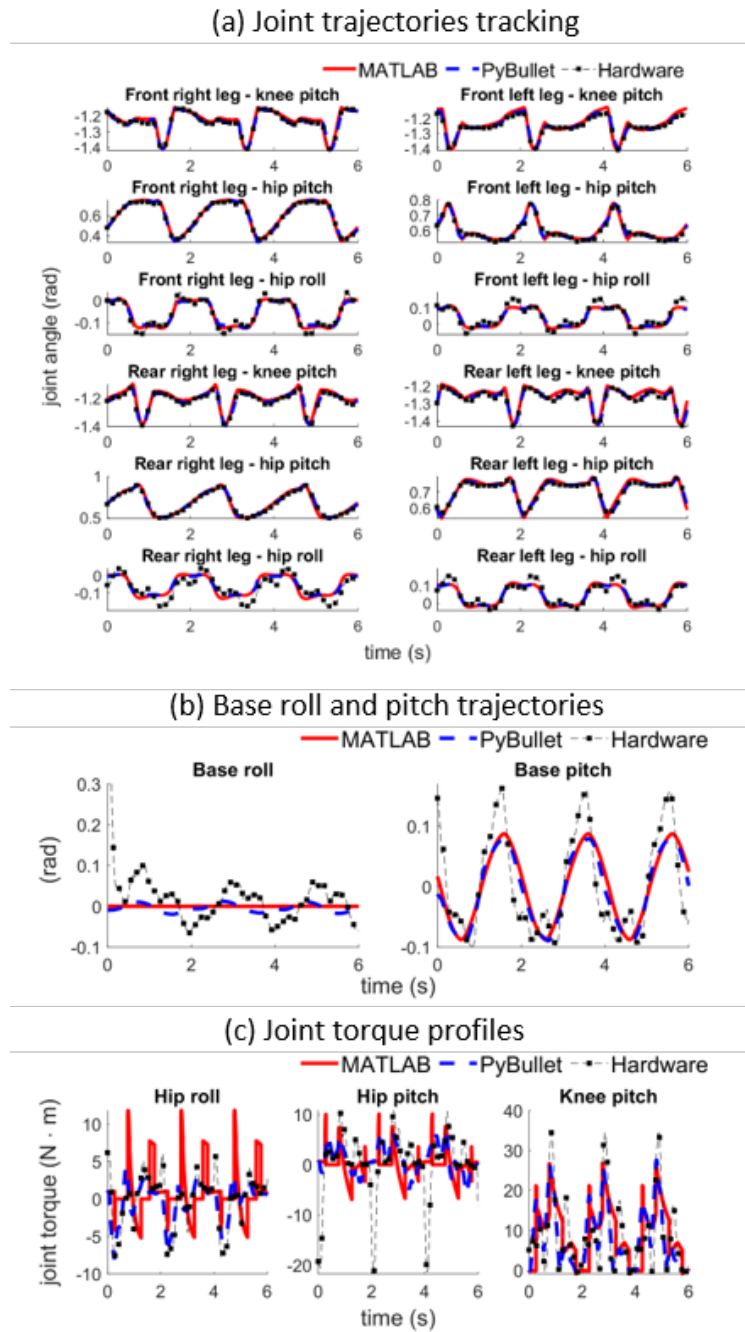


Fig. 1. Trajectory tracking results with gait (G2): (a) joint position trajectories, (b) base roll and pitch trajectories, and (c) joint torque profiles of the rear-left leg.

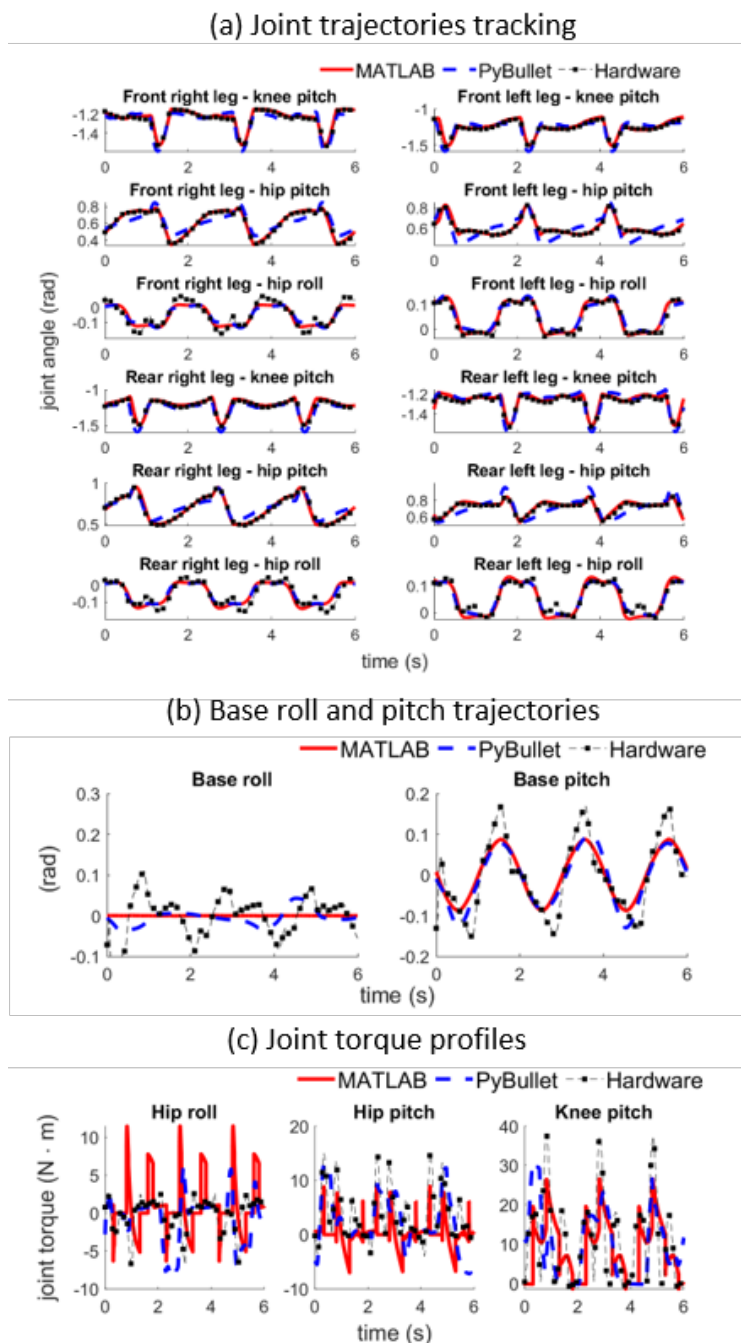


Fig. 2. Trajectory tracking results with gait (G3): (a) joint position trajectories, (b) base roll and pitch trajectories, and (c) joint torque profiles of the rear-left leg.

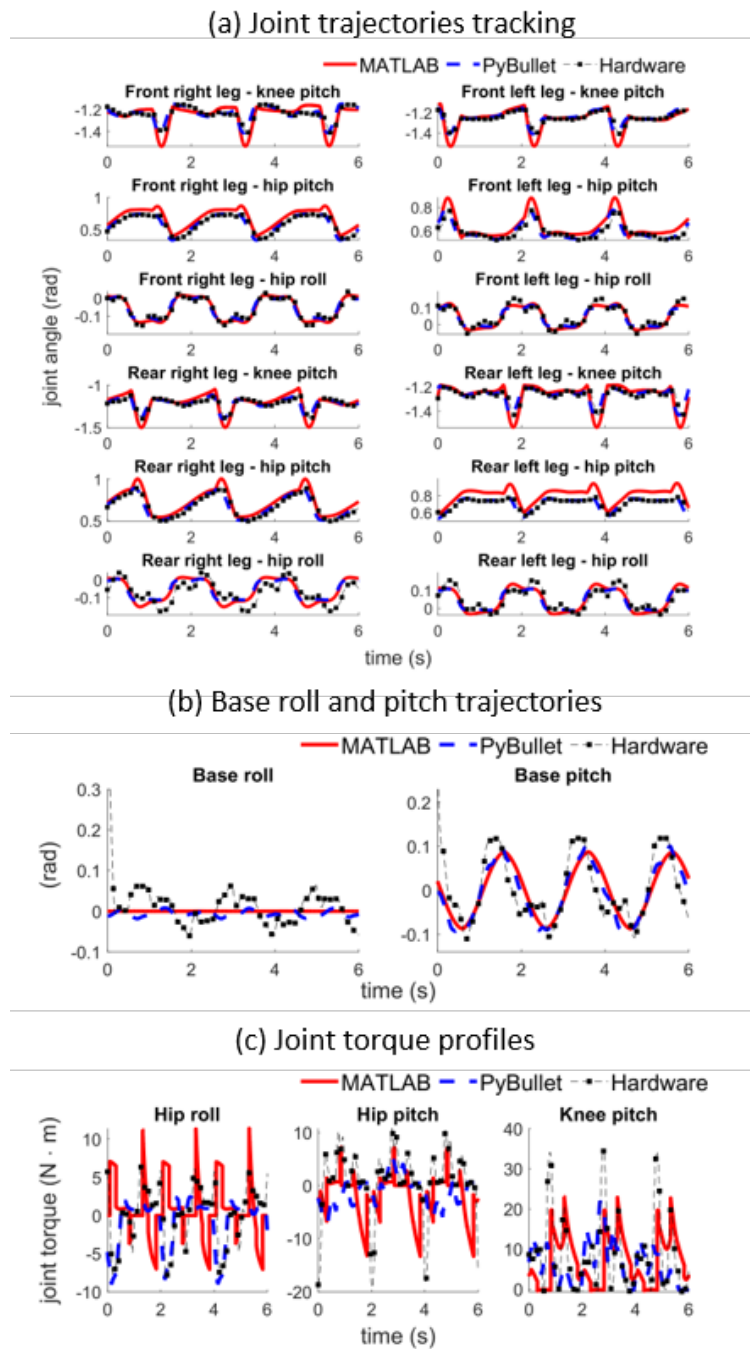


Fig. 3. Trajectory tracking results with gait (G2) under uncertainties (U2): (a) joint position trajectories, (b) base roll and pitch trajectories, and (c) joint torque profiles of the rear-left leg.

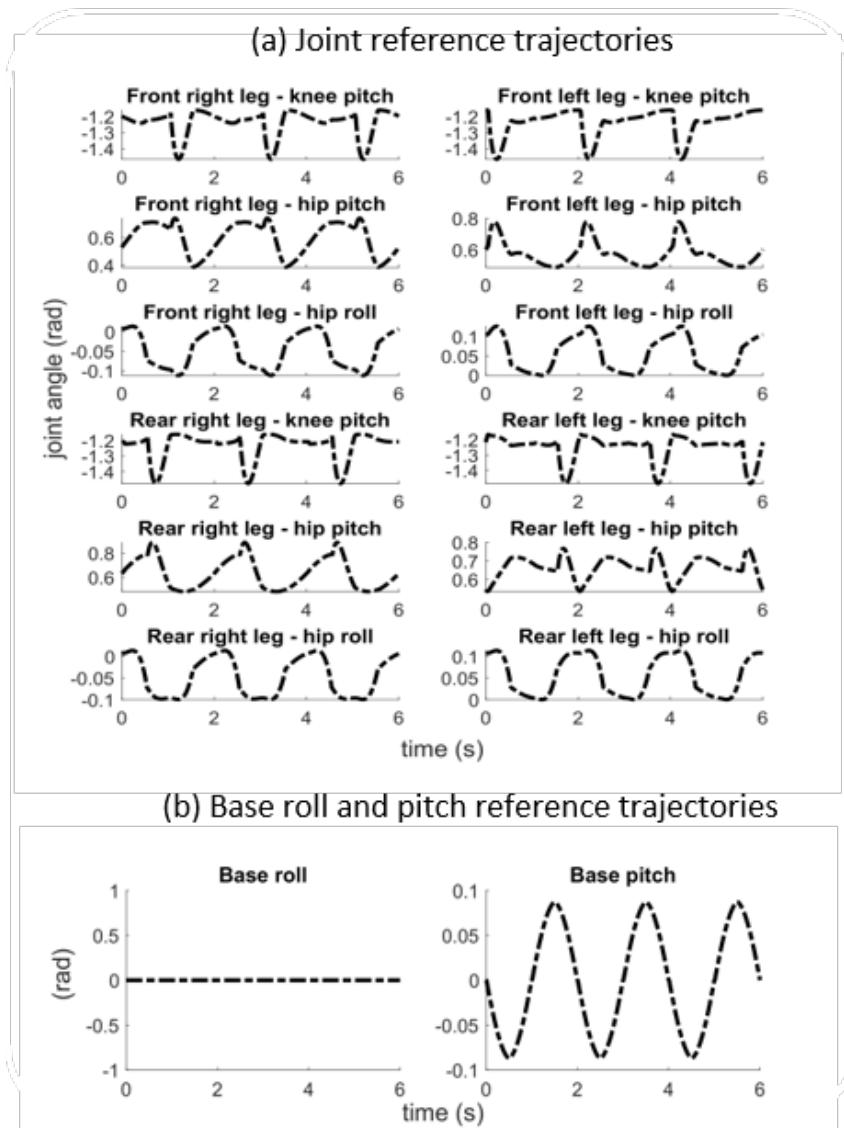


Fig. 4. Reference trajectory for the gait (G1): (a) joint reference trajectories and (b) base roll and pitch reference trajectories.

LIST OF FIGURES

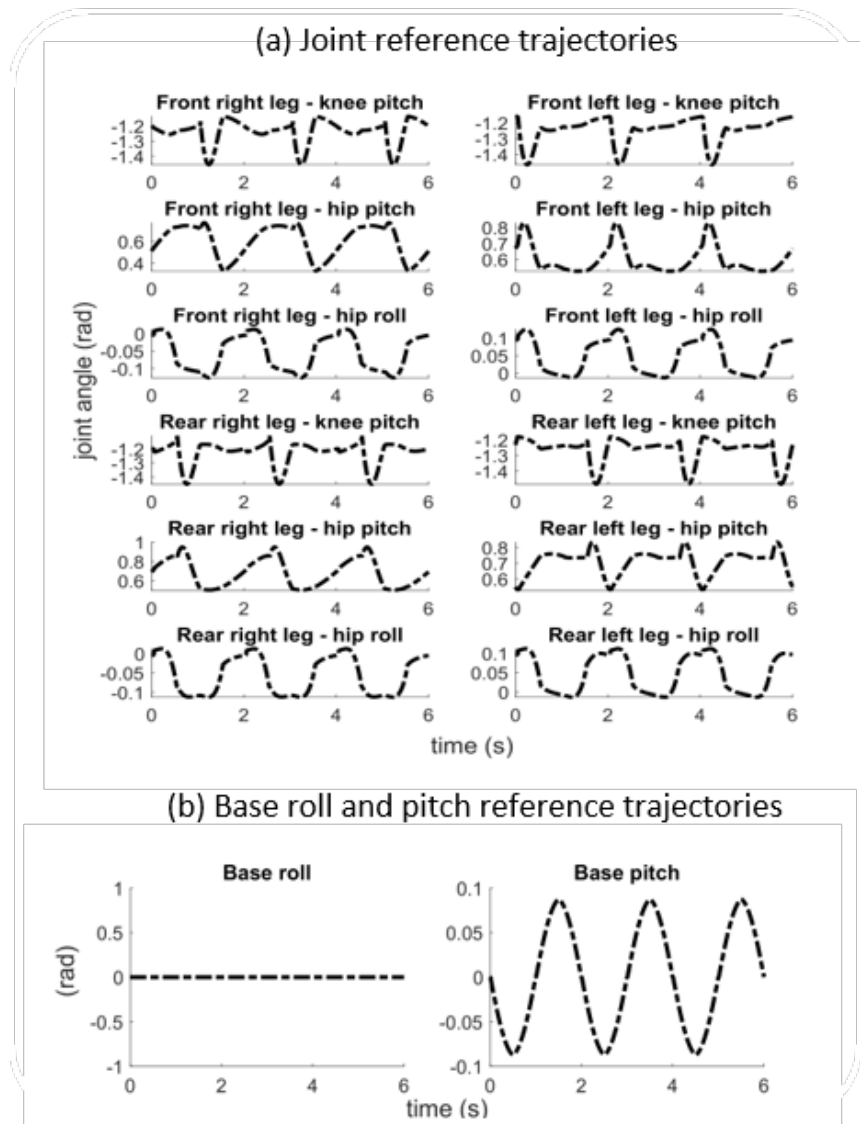


Fig. 5. Reference trajectory for the gait (G2): (a) joint reference trajectories and (b) base roll and pitch reference trajectories.

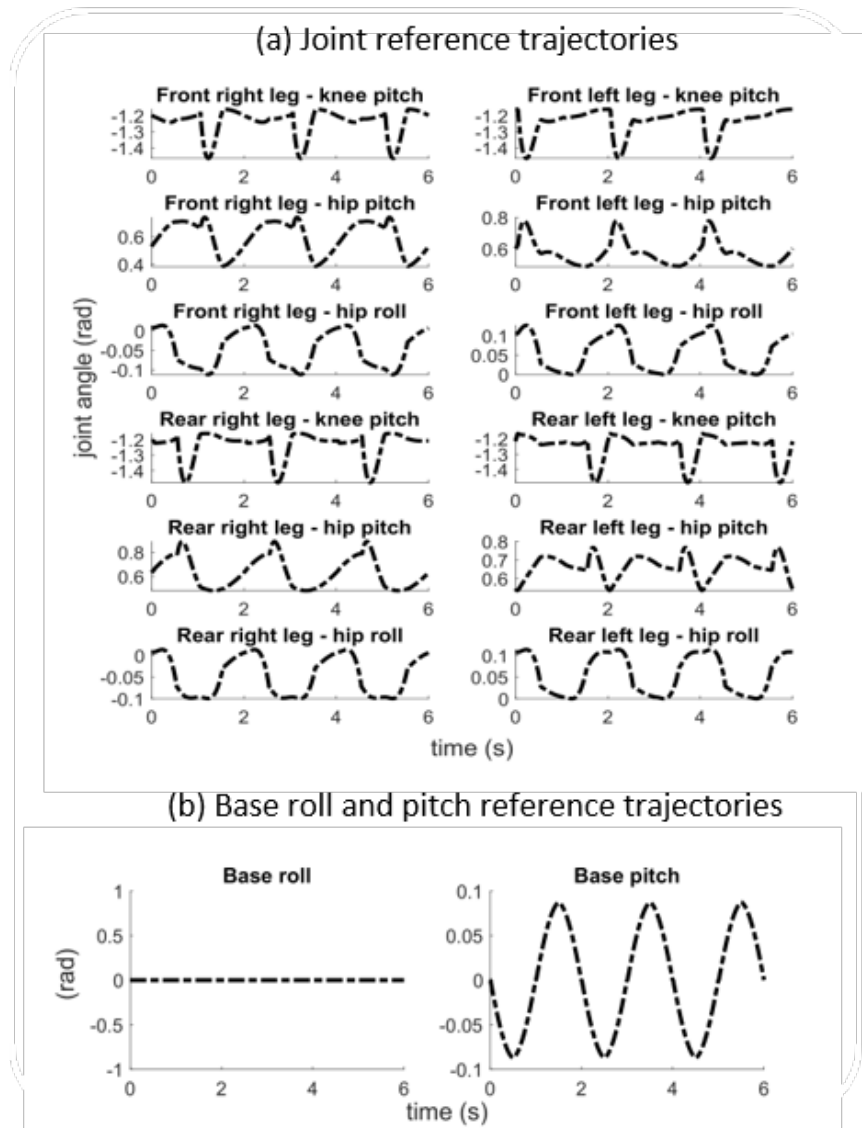


Fig. 6. Reference trajectory for the gait (G3): (a) joint reference trajectories and (b) base roll and pitch reference trajectories.