Modeling, Analysis, and Control of SLIP Running on Dynamic Platforms

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ABSTRACT

The complex dynamic behaviors of legged locomotion on stationary terrain have been extensively analyzed using a simplified dynamic model called the Spring-Loaded Inverted Pendulum (SLIP) model. However, legged locomotion on dynamic platforms has not been thoroughly investigated even by using a simplified dynamic model such as SLIP. In this paper, we present the modeling, analysis, and control of a SLIP model running on dynamic platforms. Three types of dynamic platforms are considered: a) a sinusoidally excited rigid-body platform; b) a spring-supported rigid-body platform; and c) an Euler-Bernoulli beam. These platforms capture some important domains of real-world locomotion terrain (e.g., harmonically excited platforms, suspended floors, and bridges). The interaction force model and the equations of motion of the SLIP-platform systems are derived. Numerical simulations of SLIP running on the three types of dynamic platforms reveal that the platform

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movement can destabilize the SLIP even when the initial conditions of the SLIP motion are within the domain of attraction of its motion on flat, stationary platforms. A simple control strategy that can sustain the forward motion of a SLIP on dynamic platforms is then synthesized. The effectiveness of the proposed control strategy in sustaining SLIP motion on dynamic platforms is validated through simulations.

1 INTRODUCTION

The potential capabilities of legged robots in exploring unstructured environments due to their discrete footholds have motivated extensive research on legged robotic locomotion. Since legged animal locomotion is remarkably agile and energy-efficient, researchers have proposed many simplified dynamic models called “templates” [1] to understand their underlying principles so as to inform the design and control of legged robots. A point mass at the top of a massless spring introduced by Blickhan captures important details of legged bouncing locomotion dynamics [2]. This Spring-Loaded Inverted Pendulum (SLIP) model forms an essential model for explaining and analyzing legged locomotion dynamics [3]. Since its first introduction, the SLIP model has been extensively analyzed for understanding the stability, periodicity, and energy efficiency of legged locomotion [4, 5].

Legged locomotion involves the physical interaction between a locomotor and the locomotion platform. The motion of SLIP over a rigid, stationary platform has been extensively studied [6, 7]. Schmitt and Holmes used the SLIP model to describe passive stabilization for insect running on a horizontal plane [6]. SLIP motion on uneven platforms and strategies to sustain the motion have been studied by many researchers [8]. However, analysis and control of a SLIP model on dynamic platforms have not been thoroughly investigated. Spence et al. experimentally investigated the motion of insects running on elastic surfaces [9]. Moritz et al. [10] empirically analyzed human hopping on soft, elastic surfaces. When a SLIP model interacts with a dynamic platform, the platform motion affects the SLIP motion through the reaction force at the contacting point. Thus, it is necessary to model the platform and SLIP dynamics in order to understand their interaction. The analysis of the dynamic models could be used to inform controller designs that sustain legged locomotion on dynamic platforms. This paper is structured as follows. Dynamic modeling of SLIP-platform coupled systems is introduced in Sec. 2. In Sec. 3, simulation results of the derived dynamic models are presented. The proposed control strategy for sustaining forward motion of a SLIP on dynamic platforms is covered in Sec. 4. Section 5 concludes the paper.

2 DYNAMIC MODELING OF SLIP-PLATFORM COUPLED SYSTEMS

In this section, we will derive the dynamic models of SLIP-platform coupled systems. These models will serve as a basis for the analysis and control of SLIP motion on dynamic platforms.

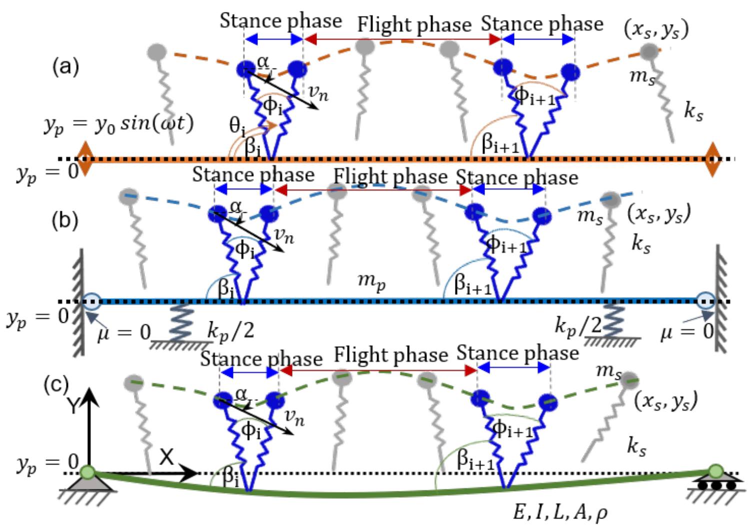

Fig. 1. Models of dynamic platforms: a) a sinusoidally excited rigid platform; b) a platform of mass (m_p) supported on spring of stiffness (k_p); and c) a simply supported Euler-Bernoulli beam with elastic modulus (E), sectional inertia (I), length (L), cross sectional area (A) and density (ρ). A SLIP model with mass (m_s) and leg stiffness (k_s) moves over these platforms. β_i is the angle of attack at the i^{th} touchdown, and ϕ_i represents the angle swung by leg during the i^{th} stance phase.

2.1 The SLIP Model

The SLIP model is the simplest model that captures the essential dynamic behaviors of legged locomotion [1, 11]. It consists of a point mass at the top of a massless spring leg. The modeling assumptions of the SLIP are listed as follows:

(A1) The spring leg is massless and can be adjusted to a desired orientation during a flight phase [4].

(A2) The entire body mass is concentrated at the hip [4].

(A3) The contact point between the SLIP and the platform does not slip, and the SLIP is confined to move in the sagittal plane [4].

The motion of the SLIP-platform system is decoupled during a flight phase and is coupled during a stance phase. Flight-phase dynamics of the SLIP can be expressed as:

$$\frac{d^2x_s(t)}{dt^2} = 0 \text{ and } \frac{d^2y_s(t)}{dt^2} = -g\ . \tag{1}$$

where x_s and y_s represent the x- and y- positions of the SLIP center of mass, respectively, and g is the gravitational acceleration. The stance-phase dynamics of the SLIP is given by:

$$\frac{d^2x_s(t)}{dt^2} = \frac{-F_x(t)}{m_s} \text{ and } \frac{d^2y_s(t)}{dt^2} = -g - \frac{F_y(t)}{m_s}\ . \tag{2}$$

where the expressions of F_x and F_y will be given in Eqs. (7) and (8), respectively, and m_s is the mass of the SLIP.

2.2 The Platform Models

Walking or running requires locomotors to interact with a platform for generating the reaction force needed to sustain locomotion. The platform can be flat or irregular, and static or dynamic. The platform types and dynamics are expected to affect the locomotion performance. For example, Alexandra et al. has shown that walking or running on uneven terrain requires significantly more energy than that on even terrain [12], which then affects the gait of a locomotor [13]. In this paper, these interactions will be analyzed based on the dynamic models of three different classes of dynamic platforms.

To represent harmonically excited rigid platforms, a platform model with the following sinusoidal displacement profile is considered (see Fig.1 (a)):

$$y_p(t) = y_0 sin\omega t, \tag{3}$$

where y_p is the transverse displacement of the platform, and y_0 and ω are the amplitude and frequency of the transverse motion, respectively.

To represent suspended locomotion platforms, a platform supported on linear, vertical springs is considered (see Fig. 1 (b)). The equations of motion for such platforms are given by:

$$\frac{d^2 y_p(t)}{dt^2} + \omega_n^2 y_p(t) = \frac{F_y(t)}{m_p}, \tag{4}$$

where m_p is the platform mass, k_p is the spring stiffness, F_y is the vertical component of the interaction force acting on the platform, and ω_n is the platform natural frequency given by $\omega_n = \sqrt{k_p/m_p}$.

To represent compliant platforms such as bridges, an Euler-Bernoulli (EB) beam [14] is considered (see Fig. 1 (c)). The dynamics of an EB beam with structural damping ignored can be described by the following 4^{th}-order partial differential equation:

$$\frac{\partial^2}{\partial x^2}(EI\frac{\partial^2 y_p(x,t)}{\partial x^2}) + \rho A \frac{\partial^2 y_p(x,t)}{\partial t^2} = F_y(t)\delta(x - x_{s,i}), \tag{5}$$

where $x_{s,i}$ is the x-coordinate of the i^{th} touchdown point on the beam, ρ, A, E, and I are the beam density, cross-sectional area, elastic modulus, and second moment of area about the axis passing through the centroid of the cross-section and normal to the loading direction, respectively, and $\delta(x - x_{s,i})$ is the Dirac-delta function defined as:

$$\delta(x - x_{s,i}) = \begin{cases} 1, & \text{if } x = x_{s,i} \ , \\ 0, & \text{otherwise} \ . \end{cases} \tag{6}$$

2.3 Interaction Force Model

SLIP locomotion on a dynamic platform involves coupled SLIP-platform dynamics during stance phases. The interaction force between the SLIP and the platform during a stance phase depends on the states of both the SLIP and the platform. The horizontal and vertical interaction forces can be respectively expressed as:

$$F_x(t) = k_s(x_s(t) - x_p(t))(1 - \frac{l_0}{l(t)}) \ , \tag{7}$$

$$F_y(t) = k_s(y_s(t) - y_p(t))(1 - \frac{l_0}{l(t)}) \ . \tag{8}$$

where x_s and y_s are the x- and y- coordinates of the SLIP center of mass, respectively, x_p and y_p are the x- and y- coordinates of the touchdown point on the platform, respectively, l_0 and l are the free and compressed length of spring, respectively, and k_s is the spring-leg stiffness. During a flight phase, the two dynamical systems are decoupled.

3 NUMERICAL SIMULATION OF SLIP-PLATFORM SYSTEMS WITHOUT ACTIVE SLIP CONTROL

To illustrate the significant effects of the dynamic properties and movement of a dynamic platform on the SLIP motion, this section presents the results of numerical simulations of the SLIP running on three general types of dynamic platforms without any active control imposed on the SLIP. The simulation results reveal that without active control a SLIP is not able to sustain forward motion across dynamic platforms in most cases.

3.1 Comparative Simulations: SLIP Running on a Rigid, Stationary Platform

For comparison, stable periodic SLIP running over a regular platform that is rigid and stationary is first simulated. The method of planning such a stable motion on regular platforms can be found in [8].

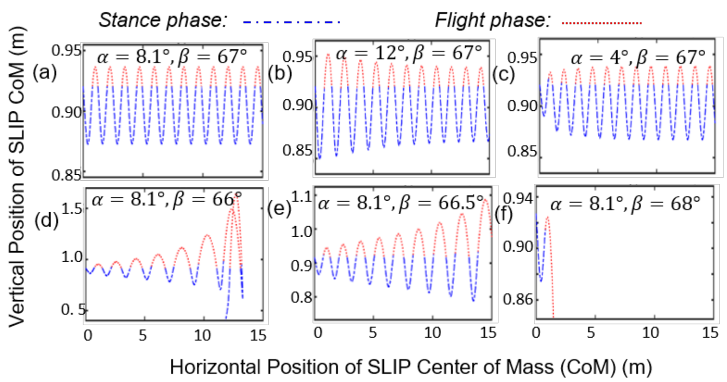

Fig. 2. SLIP center of mass motion over a flat, rigid-stationary platform ($m_s = 80$ kg, $k_s = 15$ kN/m, $\beta = 67°$, and $v_n = 4$ m/s). (a): The SLIP starts at the stable fixed point; (b), (c): The initial conditions of the SLIP are within the domain of attraction. (d), (e), and (f): The angle of attack of the SLIP is changed near the stable fixed point. These plots indicate that the domain of attraction is so narrow that even a small variation $\pm 1°$ from the nominal angle of attack destabilizes the SLIP.

The stable fixed point [4] of a SLIP model is a point in the state space that maps to itself through the Poincare return map and is also stable. The domain of attraction is the region near the stable fixed point, starting within which any state will eventually converge to the stable fixed point. The stable fixed point of the SLIP model considered in this study is calculated for regular-terrain motion, which corresponds to a touchdown speed at $v_n = 4$ m/s, an uncompressed leg length of $l_0 = 1$ m, and an angle of attack at $\beta = 67°$. The value of the fixed point is calculated as $\alpha = 8.1°$, which is the angle of the SLIP velocity at touchdown with respect to the x- axis. The domain of attraction is calculated to be between $\alpha = -15°$ and $\alpha = 18°$. In the simulations, the initial conditions of the SLIP motion are chosen to be near the calculated stable fixed point.

Simulation results under different initial conditions of the SLIP motion are shown in Fig. 2. The plots show that even a small variation (within $\pm 1°$) in the angle of attack can result in unstable SLIP motion, which indicates that without active control the motion of SLIP is very sensitive to the changes in the angle of attack even when the platform is regular. In the following subsections, SLIP running on different dynamic platforms are simulated under no active control. In these simulation, the initial conditions of the SLIP model are at the fixed point calculated in this subsection.

3.2 SLIP Running on a Platform with a Sinusoidal Motion

A platform with a sinusoidal motion (see Fig. 1 (a)) can be used to represent a class of real-world dynamic platforms that are externally harmonically excited with a fixed displacement profile (e.g., a harmonically excited floor). A distinct feature of this class of dynamic platforms is that their motions are not affected by the interaction with the locomotors as shown in Eq. (3). However, the platform motion can perturb the total energy of the SLIP through their stance-phase interactions, which can destabilize the SLIP motion. Therefore, although the SLIP starts at the stable fixed point calculated in Sec. 3.1, it may not sustain forward motion due to the destabilizing effects of SLIP-platform interaction on the SLIP motion as illustrated by the simulation results in Figs. 3 (a), (b), and (c). The platform input vibration speed determines the level of perturbation on the SLIP. For a constant amplitude, the destabilizing effect is proportional to the frequency of sinusoidal motion.

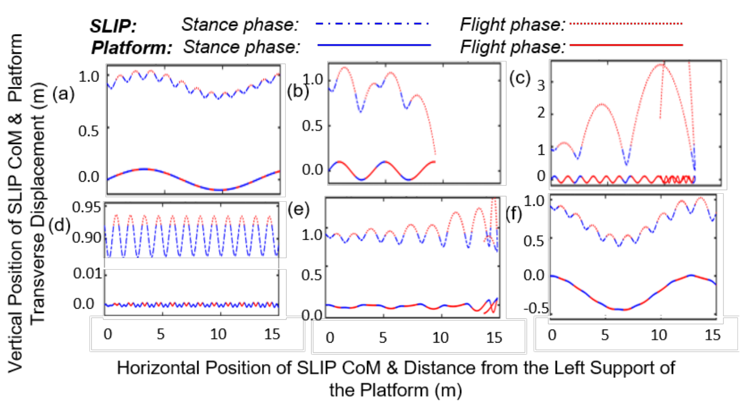

Fig. 3. SLIP center of mass motion over dynamic platforms. a), b), and c): SLIP motion over a sinusoidally excited platform with an amplitude of $y_0 = 0.1$ m and a platform frequency ($\omega/(2\pi)$) of 0.3 Hz, 1 Hz, and 3 Hz, respectively. d), e), and f): SLIP motion over a platform ($m_p = 1000$ kg) supported on springs with a natural frequency (f_n) of 10 Hz, 1 Hz, and 0.3 Hz, respectively.

3.3 SLIP Running on a Rigid Platform Supported on Springs

A rigid platform supported on vertical springs (see Fig. 1 (b)) can be used to represent a wide range of real-world compliant locomotion platforms, such as rubber tracks [15], leaf litters [9], and soft elastic surface [10]. Simulation results of SLIP running on a rigid platform with a constant mass supported on springs reveal drastically different SLIP-platform interactions under different stiffnesses of the supporting spring. Figure 3 (d) shows the simulation results of SLIP running on a stiff platform with a natural frequency greater than 5 times of the SLIP's nominal stride frequency f_0. In this case, the energy exchange between the SLIP and the platform is minimal due to two main factors: a) during a stance phase, the interaction force varies smoothly from zero (at touchdown) to a maximum value (at mid-stance) and then to zero (at take-off); and b) the duration of the SLIP's stance phase equals several displacement cycles of the stiff platform. Since the energy transferred from the SLIP to the platform during a stance phase depends on the work done by the interaction force on the platform, the above two factors result in minimal energy exchange between the SLIP and a stiff platform. Due to the minimal energy loss, the SLIP tends to preserve its motion similar to that on a rigid, stationary platform, as confirmed by the similarity between Fig. 2 (a) and Fig. 3 (d).

Figure 3 (f) shows the simulation results of SLIP running on a platform supported on soft springs with a natural frequency less than $1/5$ of the SLIP's nominal stride frequency. In this case, the SLIP takes many steps within one cycle of platform oscillation. The SLIP keeps losing energy when the platform moves downward and absorbs a portion of the lost energy back after the platform begins to move upward. If the SLIP cannot recover the lost kinetic energy, it will eventually fall down. Figure 3 (e) shows the simulation results of SLIP running on a platform supported on springs of moderate stiffness. The results depicts an intermediate effect as compared to soft (Fig. 3 (f)) and stiff (Fig. 3 (d)) platform.

The simulation results of SLIP motion on a platform supported on springs indicate that the dynamic properties of the platform indeed affect the SLIP motion. The effects on the SLIP motion depend on the platform natural frequency, which is a function of the platform mass and stiffness.

3.4 SLIP Running on an Euler-Bernoulli Beam

An Euler-Bernoulli (EB) beam (see Fig. 1 (c)) represents a class of locomotion platforms with spatially varying displacement profile. For example, bridges can be mathematically modeled as EB beams [16]. A distinctive feature of an EB beam is that its transverse displacement profile are spatially varying due to a spatially varying resistance to the applied bending force. For example, an EB beam is relatively stiff near the supporting ends and relatively soft at the mid-span. In contrast, a platform supported on springs, which is studied in Sec. 3.3, has spatially uniform stiffness and displacement profile. A SLIP-beam coupled system is simulated by numerically

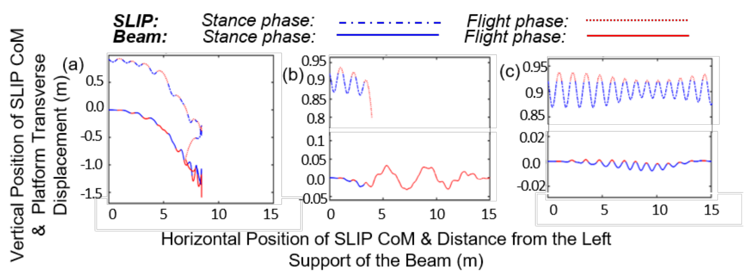

Fig. 4. SLIP center of mass motion over a simply supported Euler-Bernoulli beam ($L = 15$ m, $E = 210$ GPA, and $\rho A = 67$ kg/m). (a): $\omega_0 = 0.3$ Hz. (b): $\omega_0 = 1$ Hz. (c) $\omega_0 = 5$ Hz. ω_0 is the fundamental frequency of the beam. At higher ω_0 (i.e., $\omega_0 > 10$ Hz), the beam behaves like a stationary platform. At lower ω_0, the beam and the SLIP move together, and the passive SLIP is not able to generate enough reaction force to take off from the beam.

solving the SLIP-beam coupled dynamics. As explained in Sec. 2, the stance-phase dynamics of the coupled SLIP-beam system includes: a) the 2^{nd}-order ordinary differential equation in Eq. (2) describing the SLIP dynamics; and b) the 4^{th}-order partial differential equation in Eq. (5) describing the beam dynamics. During a flight phase, the SLIP motion is only affected by gravity, and the beam undergoes free vibration. To assess the effects of beam compliance on the SLIP motion, a group of EB beams with a relatively wide range of fundamental frequencies (ω_0) are simulated. This range is chosen based on the frequency range for different types of bridges [16, 17]. This frequency range is chosen as 0.3 Hz $< \omega_0 < 10$ Hz to include soft, moderate (1 Hz $< \omega_0 < 5$ Hz), and stiff beams. The reason for this choice of the frequency range is twofold: a) it covers the significant range of frequencies for the SLIP-platform interaction; and b) it covers the significant range of structural frequencies of short-span and long bridges.

Figure 4 (a) shows SLIP running on a soft beam ($\omega_0 = 0.3$ Hz). Similar to SLIP running on a rigid platform supported on soft springs as shown in Fig. 3 (f), the SLIP is not able to exert sufficient pushing force against the beam for taking off from the beam and initiating a significant flight phase. Instead, the SLIP follows the beam's transverse displacement profile till it eventually loses its entire kinetic energy in displacing the beam and fails to sustain forward motion.

Figure 4 (b) shows SLIP running on a moderately soft beam ($\omega_0 = 1$ Hz). Similar to SLIP motion on a moderately soft spring supported platforms (Fig. 3 (e)), the SLIP fails to move forward.

Figures 4 (c) show SLIP running on a relatively stiff beam whose fundamental frequency is 5 Hz. In this case, the SLIP is able to cross the beam, but its gait shows a significant change at the mid-span due to the large vibration speed of the beam.

The simulation results of SLIP running on EB beams highlights the spatially varying dynamic interactions between the SLIP and the beams. In all the presented cases, it is observed that the motion of a SLIP on dynamic platforms is significantly affected by the platform dynamics and that controllers will be required for sustaining forward motion.

4 CONTROLLER DESIGN FOR SUSTAINING FORWARD MOTION OF A SLIP ON DYNAMIC PLATFORMS

As discussed in Sec. 3, maintaining sustained forward motion of a SLIP on a dynamic platform is even more difficult than that on a rigid, stationary platform due to the effects of platform movement, as demonstrated by the simulation results shown in Figs. 3 and 4. The platform movement perturbs the SLIP motion and the stance-phase symmetry, which may lead to a fall. In this section, we design a simple controller that stabilizes the SLIP motion on a dynamic platform by iteratively tuning the angle of attack. The transformation from (x_p, y_p) and (x_s, y_s) to the variables β and ϕ is given as: $\beta = \tan^{-1}(\frac{y_s - y_p}{x_p - x_s})_{\text{touchdown}}$ and $\phi = (\tan^{-1}(\frac{y_s - y_p}{x_p - x_s})_{\text{takeoff}} - \beta)$. This relation is also illustrated in Fig. 5.

Angle of attack adjustment controller: This control action is designed to iteratively make the stance-phase motion near symmetric about the line normal to the platform at the contact point between the SLIP and the platform. For the considered dynamic platforms, the angle swung by

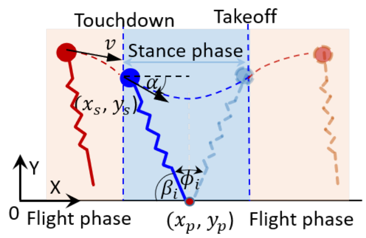

Fig. 5. Definitions of β and ϕ during a gait cycle of a SLIP

the SLIP leg during successive near symmetric stance-phases motion does not change drastically when the point mass moves with a near constant horizontal speed. The near constant horizontal speed is attained due: a) the symmetric stance-phase motion ensures that the SLIP horizontal velocity is same at a touchdown and takeoff, and b) during the flight-phase, the horizontal speed is unperturbed. An estimate of the angle swung (ϕ_i) by the leg (see Fig. 1) is used for determining the angle of attack (β_{i+1}) for the next stance phase so as to make the stance-phase motion near symmetric as explained in the Algorithm 1. This algorithm is simulated for SLIP motion on

Algorithm 1: Angle of attack control algorithm

Use hip encoder and contact sensors to measure β_i and estimate ϕ_i and initialize the control;
while *The SLIP is moving forward* **do**
  if *Stance phase is symmetric (i.e., $\phi_i = \pi - 2\beta_i$)* **then**
    $\beta_{i+1} = \beta_i$;
  else
    $\beta_{i+1} = (\pi - \phi_i)/2$;
  end
end

stationary and dynamic platforms with various initial conditions. If the SLIP moves forward after the first step, then this algorithm makes the stance phase motion near symmetric. For stationary platform, symmetry within $\pm 0.1°$ is achieved after five steps. This strategy achieves a near symmetric stance-phase motion without dissipating energy from the SLIP controller as compared to the enforced stance-phase symmetry concept introduced by Piovan et al. [7]. The designed controller is simple, intuitive, and robust with only minimal control effort. Ideally, it does not add additional energy to the system due to the massless spring assumption (A1).

Controller performance: We implemented the designed controller for SLIP motion over the considered dynamic platforms as described in Section 1.2. The motions of Section 2, Figs. 3 (b), 3 (e), and 4 (b) are simulated under the proposed controller, and the resulting motions are shown in Figs. 6 (a), (b), and (c), respectively. Simulation results confirm the efficacy of the designed controller in sustaining SLIP motion for a wide range of dynamic platforms.

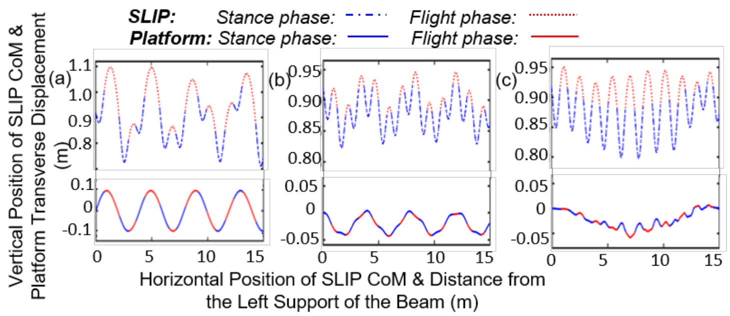

Fig. 6. SLIP center of mass motion over dynamic platforms after implementing the control strategy in simulation. a): a sinusoidally excited platform ($y_0 = 0.1$ m at 1 Hz). b): a platform supported on springs ($m_p = 1000$ kg and $f_n = 1$ Hz). c): an EB beam with fundamental frequency 1 Hz. In all these cases, the SLIP is able to sustain forward motion. Several (> 50) more variations of platforms with moderate stiffness were simulated under the proposed controller, and the SLIP was able to sustain motion in $> 90\%$ of the cases.

CONCLUSION

This paper studies the effects of platform dynamics on the locomotion performance of a SLIP model running on dynamic platforms. Three representative groups of dynamic platforms were considered, including: a) a sinusoidally excited rigid platform, b) a spring-supported platform, and c) a simply supported EB beam. Numerical simulation results of the SLIP-platform coupled systems revealed that the physical interaction between a SLIP and a dynamic platform can deteriorate the SLIP movement performance and even cause its instability in most cases. In general, the level of interaction is dependent on platform compliance and its state at the time of touchdown. For a conservative SLIP-platform system, a platform with a high compliance can rapidly destabilize the SLIP motion due to the significant energy exchange between the platform and the SLIP. To overcome the challenge of sustaining forward SLIP motion on dynamic platforms, we proposed a simple and intuitive controller. Simulation results confirmed the validity of the proposed controller for maintaining SLIP motion on the considered dynamic platforms.

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